EXPERIMENTAL RESEARCHES

ON

REINFORCED CONCRETE

BY

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TRANSLATED AND ARRANGED

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WITH AN INTRODUCTION

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INTRODUCTION.

Concrete as a material for structures, or parts of them, for the many and various needs of modern industrial life, had, as is well known, an extensive and growing application before iron or steel rods were embedded in it for reinforcing. The great advantages of concrete, viz., stability of characteristic properties, small effects caused by changes in temperature, protection against rust and heat, fireproof qualities, and, finally, the facility of adaptation to different forms and shapes, combined with a low cost of manufacture, made its still more extensive application desirable. But the resistance of concrete to the stresses and strains caused in it by external forces is low compared to that of the materials generally used by engineers, such as steel and wrought iron. Especially is the resistance of concrete to tensile and shearing stresses so small that structures or parts thereof which are subjected to such stresses to a considerable extent become uneconomical and impractical.

When it, therefore, became known from the applications made by Monier, Wayss, and others that iron embedded in concrete would act together with the latter and thus virtually strengthen it, engineers all over the world were eager to take advantage of this method of reinforcing concrete. Many and multiform applications of this principle were made and numerous letters-patent taken, each claiming superiority over the other. It is due especially to the initiative and boldness of French and German engineers and their untiring energy in overcoming difficulties and objections, both engineering and legal, that reinforced concrete has had such a rapid and successful development. Practically within the last decade reinforced concrete structures began to be universally used in

all civilized countries and to compete, in many instances successfully, with steel structures. The importance of the new material has become such that no civil engineer can well afford to be without a thorough knowledge of its properties.

As is generally the case with new materials, the practical advantages of reinforced concrete were demonstrated a long time before the theoretical analyses of its properties were attempted. Rather to explain than to study the increased resistance of concrete and steel combined, various assumptions were made as to the behavior of the new material. The most rational of these assumptions were based on the analogy of composite structures, assuming that steel embedded and well distributed in concrete will act the same as does a rod laid parallel with a piece of timber. Formulas founded on these various assumptions have been deduced, and, while they have a rational appearance, they are empirical only. As such they answer very well their purpose of giving quick and safe rules for the computation of certain reinforced concrete constructions, such as beams, especially, provided the limits of their range of application are observed. They are useful and convenient to the busy engineer in estimating and preliminary work. But it should be observed that these empirical formulas give satisfactory results only because in beams the practical proportions of depth to length vary within narrow limits and that it is comparatively easy to fit empirical constants into the chosen formulas.

This is not stated to depreciate the value of the empirical formulas, but to point out that all these coefficients of elasticity of concrete, the ratios of the latter to that of steel, the allowed unit stresses, etc., are not what they claim to be in name, but are merely numerical constants to be applied to given empirical formulas. It therefore follows that the numerical values given to the above constants cannot furnish us a true insight into the behavior of reinforced concrete.

To fully understand the action and utilize the properties of a material, its stress and strain story must be thoroughly

known and not merely results of tests of given structures. The great value of Considère's researches consists in the fact that they represent practically the first systematic attempt to study the properties of reinforced concrete in a scientific manner, by one of the world's foremost experimenters. How fruitful in results such researches can be, is proved by Considère's discovery of "hooped" concrete for compression members, and the Italian engineer Maciachini's adaptation of this principle to hooped beams.

It is in the continuation of such researches and their full discussion that the future development of reinforced concrete lies. From such investigations the designing engineer will be able to judge for himself the use he can make of this material in the vast field of modern engineering construction. This is well understood by engineers the world around, and very recently the Swiss Commission on Reinforced Concrete has decided to undertake the investigation of the subject, at a cost of about $8,000, on the lines practically laid down by Considère's researches.

The researches, the results of which are given in this book, were undertaken in the year 1898 and cover the period of time from that date to the end of 1902. The first published report of the results obtained is found in a paper by the experimenter before the French Academy of Sciences at the end of 1898. As the work proceeded M. Considère published a number of papers containing the results of his labors, the list of which is given below.* The book presented

*The following are the publications of the author on the subject:

Académie des Sciences. Influence des armatures métalliques sur les propriétés des mortiers et bétons, 12 décembre 1898 et 2 janvier, 1899.

Académie des Sciences. Variations de volume, 18 sept., 1899.

Académie des Sciences. Résistance à la traction, 18 août, 1902.

Académie des Sciences. Etude théorique du béton fretté. 25 août, 1902.

"Influence des armatures métalliques," Génie Civil, 1899.

"Résistance à la compression du béton armé et du béton fretté," Génie Civil, 1903.

"Méthode d'éupreuve des constructions en béton armé," congrés international des méthodes d'éssai à Paris, 1900.

"Contribution à l'étude des propriétés du béton armé," congrés international des méthodes d'éssai à Budapest, 1901.

here consists of a compilation of these publications arranged and classified so as to make as far as possible one coherent treatise. It has been the intent to adhere to the author's wording and treatment, avoiding at the same time unnecessary repetitions. The chapters of the book follow in general, as will be seen, the titles of the several papers as they were published and also their chronological order. Of all arrangements this appeared to be the best, containing, as each paper does, the further development of the author's views. It is hoped that this book will be found to present adequately before American engineers the famous researches of the author; and if it should, as it is hoped it will, be of any value to the engineering profession, the objects of this translation will be attained.

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REINFORCED CONCRETE.

CHAPTER I.

Reinforced Concrete in Bending.

1. The Influence of Reinforcing on Concrete.

The first attempts to reinforce concrete by embedding iron rods in it were made by practical men, not theorists, to whom much honor is due, for it is probable that theoreticians would never have advised *a priori*, a combination of materials, the heterogeneous character of which did not inspire them with confidence. With increasing boldness reinforced concrete structures have been successfully built for shops, magazines, grain elevators, water reservoirs, bridges, floors, and other structures or parts of structures. But engineers in charge of great public and private works, with some few exceptions, have hesitated to adopt the new material on a larger scale. They thought that a clearer and fuller knowledge must first be obtained of the phenomena resulting from the combination of concrete and steel and of its probable durability and resistance to atmospheric influences.

One of the most serious objections which naturally presented itself was the following: Numerous experiments made throughout the civilized world have shown that cement mortars of usual proportions cannot, when in tension, endure, without breaking, elongations exceeding on the average one hundredth of 1 per cent. of the total length of the test specimen. But when sustaining an elongation up to this limit the stress in the embedded steel does not exceed about 2,900 pounds per square inch,

that is, about a fourth or a fifth of the stress which could be allowed for the same steel in order to realize the full advantages of its use. It seemed, therefore, that the mortar or concrete of concrete-steel members, where the steel is stressed from 9,000 to 15,000 pounds per square inch, must crack, and would thus be in a very poor condition as to durability. To this objection the partisans of concrete-steel reply that the strength and solidity of their structures can be seen and that only very seldom are almost imperceptible cracks produced. But to this again their opponents answer that very small cracks escape observation, and that mortar, even without being cracked, may lose its resistance, owing to some unknown form of disintegration; and again, that while the momentary stability of a member the mortar of which is disintegrated in the portion subjected to great elongations can be explained, if the steel remains solidly embedded in the unbroken cement at the ends, no reliance can be put upon the permanent stability of such a structure if it is exposed to repeated loads. In fact, the steel near the limits of the broken portions will have to be very highly stressed, causing thereby excessive elongations, the transmission of which to the surrounding unbroken mortar must by and by extend the disintegration to the complete failure of the member. Expressions of these uncertainties are sometimes heard even from some of the partisans of the new method of construction.

The known observations and experiments do not seem to be of a character to throw sufficient light on the subject. Observations have been generally limited to the measurements of the deflections of beams subjected to loads; but it is wholly impossible to deduce from them the elongations of the "fibres" in tension and the shortenings of the compressed "fibres" produced after the elastic limit has been exceeded at any point. Sometimes, it is true, these deformations have been measured, but mostly

the intention was to determine the elongation of the steel itself. To reach the steel with the measuring apparatus cuts were made in the concrete which destroyed its continuity and prevented getting from the observations a clear picture of the phenomena which really take place in beams having no cracks.

In some cases the accuracy of the measurements of the loads employed is also of a doubtful character. When the loads have been made up of numerous bags of sand piled on the beams to a height of 5 to 7 feet, arch action was produced which must have relieved the middle portion of the beams to a noticeable degree. With loads produced by means of a hydraulic press errors from manometer readings could occur, and it seems that such was the case with some experiments where, contrary to what has been found elsewhere, observations seemed to indicate that the deformations increase more and more slowly as the stresses increase.

For these reasons, and also because no sure deductions can be based on experiments whose details were not followed carefully, the author decided to make new tests. Since it was clearly necessary to have numerous tests, small prisms only were used. In doing so the author was guided by the consideration that the laws governing the materials would be shown in small prisms, and could then be verified by a small number of tests on large prisms so arranged as to furnish directly the necessary data. It appeared to be proper to abandon the measuring of deflections which to the other complications of the action of bodies in flexure adds one more, because the deflection is the resultant of different deformations produced in all the sections of the member in flexure. The author, therefore, adhered exclusively to the measuring of the elongations of the surfaces in tension and the shortenings of the compressed faces. In order that these deformations be uniform for the whole length between observation

marks it was necessary that the prism be subjected to a constant bending movement. These several conditions were fulfilled by adopting the following arrangement:

Prisms 23.6 inches long by 2.36 inches square were moulded, some without reinforcing for the determination of the properties of the mortar, independent of the steel, and others reinforced on the tension side only by wires or rods of steel or iron varying from 0.075 to 0.303 inches diameter. The greatest number had no transverse reinforcing in order to simplify the observed phenomena. Each prism was placed vertically with its lower end fixed in a solid testing block, Fig. 1, and the upper end

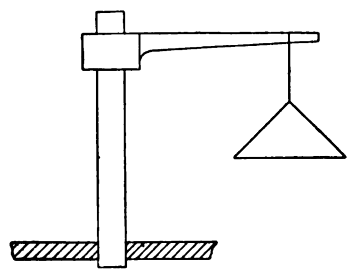

FIG. 1.

provided with a cap which also formed a fixed attachment and had a horizontal lever 27.5 inches long suspending a scale. In the fixed portions shearing stresses of varying intensity were produced by changing their lengths and their distances from the ends of the prisms. In the portion between the fixed ends, there was no shearing stress and the simple phenomenon of flexure could be observed.

These experiments were not completed at the date of writing, but the results already obtained will be described and their practical consequences indicated. Among the numerous tests, all showing concurring results, the follow-

ing gave particularly precise data and is chosen for a complete statement.

From mortar mixed 730 pounds of Portland cement and 4.3 cubic feet of water per cubic yard of good quartzy sea sand, were made one prism, No. 31, not reinforced, and five prisms reinforced by 17 wires of 0.075 inch diameter, by three wires of 0.167 inch diameter, or by iron rods 0.303 inch diameter. The author himself tamped the prisms with exceptional care. Prism No. 31, with a section 2.36 inches deep and 2.4 inches wide, broke after having resisted for several minutes a bending moment of about 83 foot-pounds, which produced a shortening of the extreme compressed "fibres"

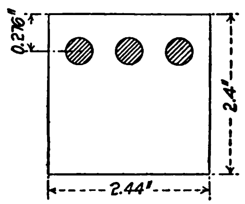

FIG. 2.

of 0.0131 per cent. and elongation of the "fibres" in tension of 0.0201 per cent. of the length. The mirror index used to show elongations by its displacements could alone be followed by the eye until the rupture, which took place after an elongation of 0.0266 per cent.

Prism No. 34 had the section shown in Fig. 2. The wires had a diameter of 0.167 inch. Only the values for this prism will be given in detail, but the results of the other prisms reinforced by wires 0.167 and 0.075 inch diameter were almost absolutely identical. The results of the prism reinforced by a rolled rod 0.303 inch diameter differed only in the smaller elasticity of rolled iron and the greater distance from the centers of the rods to the surface of the prism, made necessary by the larger

diameter. Prism No. 34 was kept in water 210 days and tested 3 days after it was taken out. Its bending moment was increased to about 569 foot-pounds without causing failure. Afterward, to study the effect of repeated deformations, this prism was subjected to 139,052 repetitions of the bending moment, varying from 250 to 402 foot-pounds, alternating with the same number of returns to the position of equilibrium. After this double test the prism appeared intact for the whole length between the fixed points, although the mortar had sustained, during the first bending, an elongation of 0.198 per cent., that is, almost twenty times as much as the 0.01 per cent. which similar mortars cannot sustain without breaking, and afterward withstood 139,052 elongations varying between 0.0545 and 0.127 per cent. To determine whether the elongated portions of the mortar were cracked the author detached this mortar, by means of a sand saw, from the rods and the body of the prism, and found that it remained perfectly intact except for two superficial cracks 0.08 to 0.16 inch long. Notwithstanding the strain due to sawing, pieces 0.59 by 0.47 inch in section and 3.15 to 7.9 inches long, that is, more than half the length between supports, were cut off, and, tested by bending, gave resistances up to 313 pounds per square inch, thus proving that, almost throughout, the surface layers, which had sustained an elongation twenty times that considered dangerous, not only were not disintegrated, but retained nearly the same strength as new mortar.

The total resistance of a reinforced prism cannot be anything but the sum of the resistances of its two elements. The elongation of the steel can be computed from the deformations as measured on the two opposite faces of the prism, making the classical assumption of the theory of flexure that the cross-sectional planes remain planes after bending; and the tension in the reinforcing bars can be easily deduced when the coefficient of elas-

ticity of the steel is known. In this case the coefficient was established by a direct tension test on an identical test piece. Multiplying the tension in the reinforcing members by its lever arm, that is, by its distance from the resultant of the mortar in compression, a portion of which forms a couple with the tension in the bars, the moment of the couple is obtained. (See Table I, column 10.) Subtracting this moment from the total bending moment gives the moment of the mortar in tension and of that fraction of the resultant of the mortar in compression, which forms a couple with it. Table I shows the results obtained.

In column 11 the fourth and the last values are unexpectedly high, and the author first attributed this to errors of observation. But the same anomaly was observed on similar prisms and is, probably, satisfactorily explained by the longitudinal sliding of the rods in the mortar. The moment of resistance of the mortar tension increased with the deformation, first rapidly and uniformly, then more and more slowly up to 1,400 inch-pounds, and did not fall below this amount afterward, though the elongation at the tension-face reached 0.198 per cent. Column 11 suffices to prove that mortar reinforced by metal can sustain elongations very much greater than those allowed for it up to the present and still contribute efficiently to the resistance of the member.

At first sight this result appears inadmissible, but the objection vanishes when one remembers the phenomena of the deformation of metals and admits that identical phenomena can take place in mortars, though the extreme smallness of the deformations has precluded their observation. When a round rod of soft steel, for instance, is subjected to simple tension, it first undergoes a uniform elongation, increasing to 18 to 22 per cent., and then it suddenly strangles itself at the "point of stricture" until rupture ensues after a local elongation of 200 to 300 per

TABLE I.

Bending Moment Supported by Beam.	Distance from Neutral Axis to Surface.		Elongations.		Value of E for Steel.	Tension in Steel.		Lever Arm on Tension Side.	Moments Caused by Steel.	Moments Caused by Concrete; Difference Between Cols. 1-10.
	Compression Side.	Tension Side.	Measured on Concrete.	Computed for Steel.		Per Square Inch.	Total.			
1	2	3	4	5	6	7	8	9	10	11
Inch-lbs.	Inches.	Inches.	In one Th	ousandths	Lbs. p. sq. in.	Pounds.	Pounds.	Inches.	Inch-lbs.	Inch-lbs.
450	1.13	1.27	0.038	0.031	30,860,000	955	62	1.77	110	340
995	1.13	1.27	0.092	0 075	30,860,000	2,320	152	1.77	296	699
1,737	1.13	1.27	0.186	0.145	30,860,000	4,485	205	1.77	522	1,215
2,640	1.08	1.32	0.424	0.337	30,580,000	10,310	677	1.77	1,200	1,440
3,555	1 01	1.39	0.775	0 620	30,010,000	18,650	1,225	1.75	2,145	1,410
4,280	1.00	1.41	1.050	0 840	29,870,000	25,050	1,653	1.74	2,880	1,400
5 550	0 96	1 45	1.520	1.230	29,300,000	36,050	2,375	1.73	4,110	1,440
6,830	0.96	1.44	1.980	1.600	28,446,000	45,500	2,990	1.73	5,170	1,660

cent. This takes place when a new elongation causes an increase in the unit stress of the deformed section which is not commensurate with the reduced section. The author does not believe it to be necessary to reproduce here the complete theory of stricture. (See paper by M. Considère, in "Annales des Ponts et Chaussées," 1885.)

In steel bars subjected to bending stricture does not take place as in tension members, because the swelling of the compressed fibres compensates the narrowing of the fibres in tension, and also because the fibres in tension do not all reach the critical elongation at the same time. It seems evident that stricture will not be produced in a round bar of soft steel completely united to a parallel bar of a metal of much higher elastic limit. When subjected to tension each increase in elongation causes a considerable increase in the resistance of the high tension bar at the instant when the soft steel will tend to produce stricture. In consequence of its connection with a more resistant metal the soft steel will have to elongate uniformly in all its parts and its average elongation, measured between any two test marks, will, therefore, be equal to the greatest molecular elongation of which it is capable, that is, 200 to 300 per cent. An identical bar, but not reinforced by a higher grade metal, will, at the same time, be strangled in a short length and will show an average elongation of 18 to 22 per cent. only. Only the study of experimental data can determine whether mortar possesses similar properties. Mr. Debray made exact experiments on the subject at the laboratory of l'Ecole des Ponts et Chaussées, and the author made many tests on mortars containing 660 to 1,500 pounds of cement to the cubic yard of sand and on prisms of neat cement. The average result was that the elongation of the mortar is about two and one-half times greater in bending than in tension.

The tests on prism No. 34 prove that the elongation of mortar in bending, though greater than in tension, is yet

far from its real ductility, since the mortar of this prism was elongated eight times as much, that is, about 0.2 per cent. It must, therefore, be admitted that the support which the less stressed "fibres" in tension or those in compression give to the more stressed and elongated "fibres" is not sufficient to develop fully the ductility of the mortar. The latter will appear when the mortar is associated with a metal having an elastic limit considerably exceeding its own, which will consequently contribute materially to the support of the weakest sections, stopping their premature deformation and making each section take the greatest elongation of which it is capable. The reinforcing metal does not change the intrinsic properties of the mortar or concrete, but enables them to produce their resistances simultaneously in all sections, increasing thereby the strength and durability of the structure.

When reinforced concrete reaches in all sections its greatest elongation it also develops its greatest resistance in all sections. But concrete is not a homogeneous material, and it is quite certain that if the action in each section could be observed the corresponding moments of resistance of the mortar would be found appreciably different. These inequalities compensate the inequalities in the stresses in the metal. The values of moments developed by mortar in reinforced prisms are computed from the average value of the deformations of all the sections between the gauge lines and can, therefore, only represent *average values.* These values must then necessarily be higher than the breaking moments of prisms made of the same mortar, but not reinforced, which break when subjected to a stress equal to the resistance of the *weakest section.* These considerations explain why in prism No. 34 the mortar developed a moment exceeding 1,400 inch-pounds, while prism No. 31, made from mortar of the same batch, broke under a moment of about 1,000 inch-pounds. It is probable that if all the sections of

prism No. 31 could have been broken successively some of them would have sustained moments exceeding 1,400 inch-pounds, so as to give an average much higher than 1,000.

The study of the results given above makes possible the determination of the coefficient of elasticity and the tensile and compressive resistances of the mortar when it undergoes successive deformations. By means of these elements the curve of deformation can be plotted with the elongations and shortenings for positive and negative abscissas and the tensions and compressions for positive and negative ordinates. The coefficient of elasticity under low stress is given by the inclination of the tangent to the X-axis at the origin of the co-ordinates. In the present case it can be easily deduced from the test of prism No. 31, in which the deformations of the mortar were not hindered by any reinforcing bars. In the bending test of this prism, measurements were begun under a bending moment of 50 inch-pounds, and the application of a supplementary moment of 400 inch-pounds produced an elongation of 0.0045 per cent. on the tension side and a shortening of 0.0036 per cent. on the compression side. Hence, under a light load the coefficient of elasticity was 3,470,000 pounds per square inch in tension, and 5,530,000 pounds in compression. Without going into the considerable effect of tamping on the properties of mortar and concrete, it may suffice to say that the difference between the coefficients of elasticity in tension and compression in prisms Nos. 31 and 34 is merely due to a difference in the compactness of the two sides, and that several experiments have proved that the coefficients are absolutely or very nearly the same for identical mortars as long as the stresses are of low intensity.

The tangents, at the origin of co-ordinates, to the curve of deformation of the mortar of prisms Nos. 31 and 34 being known, it can be seen that by successive

trials the several points of the curve can be found by finding the successive values which must be given to the tension and compression to obtain the resisting moments which the mortar has developed in prism No. 34, given in column 11 of Table I. The computed values of these tensions and compressions are given in Table II.

TABLE II.

Elongations of mortar in parts of 1 per cent....	0.004	0.010	0.025	0.050	0.100	0.150	0.198
Corresponding tensions in pounds per square inch.	138	228	256	299	300	302	303
Shortenings of mortar in parts of 1 per cent....	0.004	0.010	0.025	0.050	0.100	0.128	.....
Corresponding compressions in pounds per square inch	222	498	939	1,540	2,520	2,940	.....

It is seen that the compressive stresses grow less and less rapidly as the shortenings increase, but the change in the elasticity is much more considerable for the tensile stresses. Beyond a certain limit it even seems that the tension does not sensibly increase with the increase in elongation. The phenomenon of flexure is very complex, and the figures of Table II, which are deduced from it, cannot be considered as strictly exact. The author estimates the possible errors at about 5 per cent. Greater precision will be obtained from simple tension tests of reinforced specimens. Although perfect accuracy is desirable from the scientific point of view, it is less important for practical purposes because these materials are so heterogeneous. In prism No. 31 the coefficient of elasticity of the upper side, in moulding, was 60 per cent. greater than that of the lower side. If, therefore, one desires to compute the dimensions required for a reinforced prism either numerous experiments on the concrete to be used will have to be made, or the figures given by experimenters and based on numerous experiments on prisms of ordinary proportions and made without special care will have to be adopted.

2. Results of Known Experiments.

The laboratory tests on prism No. 34 were of a scientific character, intended to furnish information on a molecular property of mortar. This prism was consequently made with exceptional care, which cannot be expected in practice. To obtain information on which rules for designing can be based, observations must be made on the actual construction and behavior of structures. As stated the majority of recorded observations were made under conditions which do not allow very precise conclusions. However, it has been clearly established that generally the first cracks have not appeared before the bending moment acting on the highest stressed section caused tensile stresses in the iron from 23,000 to 28,000 pounds per square inch and, therefore, elongations of about 0.1 per cent. of the length of the test piece. The author has attempted to prove that if concrete-steel can endure much greater elongations than unreinforced concrete, it is because the steel efficiently aids the sections which show a tendency to deform more than the others and thus compels the concrete to take all the molecular elongation of which it is capable over the whole length of its extreme "fibres." The aid given to the concrete by the steel is evidently proportional to the coefficient of elasticity of the latter; but this coefficient drops suddenly to one-tenth and sometimes even to one-fiftieth of its initial value when the elastic limit is exceeded.

From the point of view of the protection given the concrete against premature cracks, the passing of the elastic limit produces the same result as if the sectional area of the steel were reduced to one-tenth or one-fiftieth, which then becomes negligeable. The concrete must necessarily crack in the weakest sections as soon as the steel is stressed above its elastic limit. It is impossible for concrete to sustain without cracking an elongation ex-

ceeding about 0.1 per cent. in prisms reinforced by wrought iron or extra soft steel. If in the prism No. 34 an elongation of nearly 0.2 per cent. was obtained, it was because the iron wire had an elastic limit exceeding 54,000 pounds.

It was seen in section 1 that the study of the curve of the bending moments added a new proof to the results of the direct examination of the mortar cut from prism No. 34. It confirmed that the mortar was not broken before the elongation of 0.2 per cent. was reached, since it maintained until then a resisting moment of great value. It is a matter of research to discover if well-known experiments will permit a verification of this phenomenon. This can be easily done even when no information is at hand as to the quality and resistance of the concrete and steel used in the beams whose successive loadings and corresponding deformations are known. If the result of the displacement of the neutral axis, which is of small importance, be neglected, it may be said that the bending moments induced by the tension of the reinforcing metal are very closely proportional to the deformations. If, therefore, a curve be plotted with deformations for abscissas and bending moments for ordinates, the moments caused by the metal will be represented by a straight line, O F beginning at the origin of co-ordinates O, as long as the elastic limit of the metal is not exceeded. (See Figure 3.) The total moments, being the sum of the moments of the steel and the concrete, would give a discontinuous curve, partly concave, such as O A B C F if the concrete break under a small elongation such as

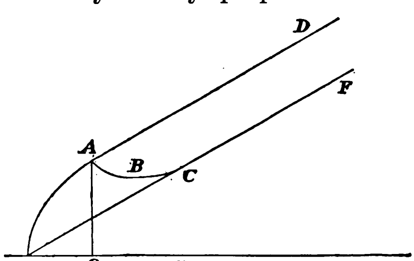

FIG. 3.

0.01 per cent., causing the rupture of unreinforced concrete. If, on the contrary, the concrete has neither disintegrated nor noticeably cracked, the elastic limit not having been reached, and has produced a bending moment which increases rapidly up to a value below which it does not fall, the curve of moments must take a form like O A D, first convex, and then practically straight. But this is exactly what is shown by all known experiments, and especially those made at Lausanne, Switzerland, by M. Ferrari, in 1893 and 1894.

3. The Resistance and Elasticity of the Materials Generally Used.

Examined in line with the above ideas the results of the experiments known to us seem to confirm the conclusions drawn from the tests on prism No. 34 and from all the rest of the author's experiments. A commonly used concrete is made of 550 pounds of Portland cement to one cubic yard of a mixture of equal parts of sand and small gravel. This concrete often gives a tensile resistance of 215 pounds, and more than 2,600 pounds per square inch in compression, with an elastic coefficient of 2,800,000 to 3,700,000 pounds; but the author would adopt, for the sake of prudence, as the least and safe values, 170, 2,140, and 2,700,000. These values give for the curve of deformation the following figures:

TABLE III.

Elongations or shortenings in per cents of length..	0.004	0.010	0.025	0.050	0.100	0.150
Corresponding tensions in pounds per square inch.	107	156	170	170	170	170
Compressions in pounds per square inch	107	256	568	925	1495	2140

4. The Graphic Solution of the Problem.

Plotting the elongations or shortenings as abscissas and tensions and compressions as ordinates the curve of de-

formation in Fig. 4 is obtained, from which by trial computations the stresses at all points of the concrete and steel of a section of a concrete-steel beam can be determined, if a known deformation is given to any of the fibres. Suppose, for instance, that the bending moment produced in a prism made of the above concrete

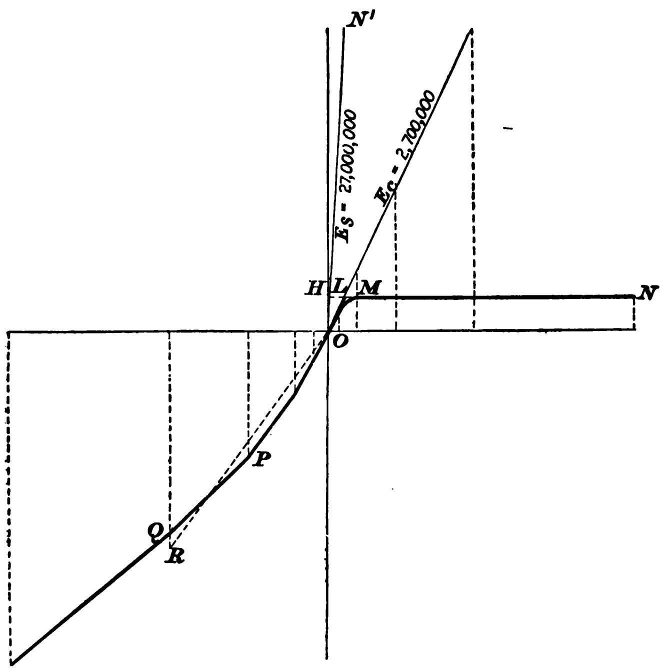

Fig. 4.

is to be determined when the reinforcing steel is stressed somewhat below the elastic limit, say at 23,000 pounds per square inch, and will take an elongation of 0.09 per cent. In Fig. 5, let A B represent the depth of the cross-section of the prism and F the center of gravity of the reinforcing metal. Let an abscissa F f be drawn at F rep-

resenting an elongation of 0.09 per cent. If the usual hypothesis of the conservation of the planes of the sections after bending is accepted as practically accurate, section A B will after bending take the position A′ B′, which must pass through the point f for steel stressed at 23,000 pounds per square inch.

The position A′ B′, as yet unknown, must satisfy the condition that the sum of the tensions in the metal and concrete in the portion O B must be equal to the sum of

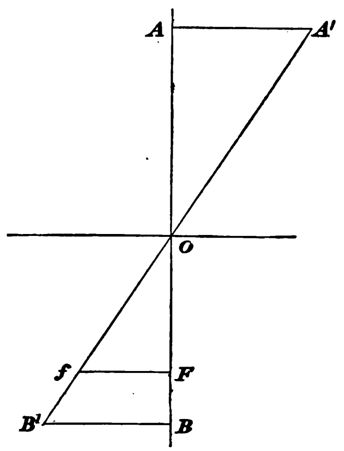

FIG. 5.

the compressive stresses of the concrete in the portion O A. To test this let us try a direction A′ f B′. In each "fibre" of section A B the elongation or the shortening will be equal to the corresponding ordinate of the straight line A′ f B′. By scaling from the curve of deformation, Fig. 4, the ordinate corresponding to each elongation or shortening, the value of the tension or compression which it will cause will be found. It will be easy to compute graphically the sums of the tensions and compressions in all the "fibres" of the concrete and metal and

to see whether the two sums are equal, and thus determine whether the trial direction of A′ B′ satisfies the condition of equilibrium. When the correct position of A′ B′ is found, the intersection of A B and A′ f B′ will give the position of the neutral axis O, and it will be easy to determine graphically the bending moment caused by the tensions and compressions in the metal and concrete of the prism.

5. The Approximate Solution.

The preceding method gives an exact solution; but if the irregularity of the properties of concrete be taken into account, it seems a waste of energy to try to obtain absolute accuracy in the methods of computation of the dimensions of the members, and so an approximate and rapid method will be given. By substituting for the curve of elongation of the concrete, O M N in Fig. 4, the horizontal line H N, which coincicdes with the greatest portion of this curve, only a very small error will be committed as far as the computation of the moments is concerned, since the triangular space O H M has a small area and a very much reduced lever arm. Similarly the straight line O R can be substituted for the curve O P Q. Equations can be now established and solved directly, but before proceeding, the object to be attained must be precisely stated.

The danger which threatens the structure if the stress in the metal exceeds the elastic limit has been stated. The greater or less elongation to which concrete is subjected within the limits of practice does not appear to be of great importance, because the tension does not vary correspondingly and because rupture depends much less upon the intrinsic value of the elongation than upon the passing of the elastic limit of its reinforcing members. It thus appears that, except the elastic limit of the metal, nothing but the crushing resistance of the concrete in

compression needs to be considered *as far as the resistance to bending is concerned.*

A third important factor to be considered is the sliding of the reinforcing metal in the concrete, but this factor has no direct relation to the value of the bending moment and depends solely on the shearing force. Hence it can be left aside while considering the bending effect and be investigated separately. To develop the method for computing algebraically the compressive stress induced in the concrete of a given prism when the reinforcing metal has reached the elastic limit, and determining the moment of bending resistance which is then produced by the prism proceed as follows: Denote by h the depth and by e the width of a cross-section of the prism in inches; by p the percentage or ratio of the area of the metal reinforcing to the area of the concrete; by hu the distance from the center of gravity of the reinforcing bars to the extreme "fibres" in tension; by l the elastic limit of the metal; by t the stress caused in concrete in tension when its elongation reaches 0.015 to 0.020 per cent., and which then remains constant with an increase of deformation; by c the compression in the most compressed "fibre" of the concrete; by K the ratio of the angular coefficient of the line O R, Fig. 4, which represents the average coefficient of elasticity of the concrete in compression, to the angular coefficient of the line O N′, which represents the coefficient of elasticity of the metal. If we assume the average coefficient of elasticity at 28,000,000 for steel and at 2,800,000 for concrete, K will have the value of 0.1 during the perfectly elastic state and decreasing values below 0.1 as increased deformations of the concrete in compression cause a greater change in the elastic behavior.

The position of the neutral axis must be determined first. If hx denote the distance from the neutral axis of the prism to the extreme "fibres" in tension, the value of x

will be obtained by equating the tensions in the steel and concrete to the compressions in the concrete. The value of the greatest compression of the concrete is given by the formula, in which

$$c = Kl \times \frac{1 - x}{x - u} \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

The value of x which gives the ratio of the distance from the neutral axis to the extreme "fibres" in tension to the total depth h of the prism then follows from the above condition:

$$t\,x + l\,p = \frac{K\,l}{2} \cdot \frac{(l - x)^2}{x - u} \dots\dots\dots\dots\dots\dots (2)$$

The moment of resistance to bending is given by the formula:

$$M = e\,h^2\left(t\,x\frac{4 - x}{6} + l\,p\frac{x - 3\,u + 2}{3}\right) \dots\dots\dots (3)$$

When the quality of the concrete and metal to be used are selected the values to be given to l, t and K will be known. For $h\,u$ the most suitable value will be chosen, and it will generally be practically the same in all cases, namely about 0.12 h. For the ratio of steel to concrete sections different values such as 0.01, 0.02, 0.03, 0.04 will be assumed. The equations will then be solved for each value of p and, by interpolation, there will be found for any value of p the position of the neutral axis, the amount of the greatest compression in the concrete and the moment of resistance to bending when the stress increases until the elastic limit of the metal is reached.

In a very short time a table can be computed which will furnish in two or three minutes the bending moment which a prism of any dimensions and percentage of metal will be able to sustain. Table IV has been computed for a beam of one inch square section. For the first three cases a concrete poor in cement and for the rest a rich concrete was assumed. To find the moment which a

prism of height h and width e will sustain the figures in column 11 of table IV have to be multiplied by $e\,h^2$ expressed in inches. The quality of concretes is much too variable to ascribe to the figures of columns 5 and 6 any general applicability. They are simply illustrative of a method of computation the results of which will be interpreted in each case according to the quality of the materials to be used. Intentionally the author has taken values below the average.

6. The Displacements of the Neutral Axis.

Column 9 of Table IV shows a series of displacements of the neutral axis under eight assumed different conditions. It gives the variations of the distance of the neutral axis from the extreme "fibre" of the tension side expressed in parts of the depth of the cross-section. The values which are computed for the tension and compression in the concrete and metal depend upon this distance to the neutral axis, no matter which method of computation may be used. If the first three lines of Table IV be examined it will be seen that the distance from the neutral axis to the tension face is reduced from 0.57 to 0.42 of the depth when the percentage of the metal to the concrete increases from 1 to 3 per cent. The area of the concrete in compression increases with the area of the metal, the tensile stress of which it must balance. Not only does the neutral axis occupy different positions in beams of different character, but in the same beam it very frequently shifts in a very noticeable manner during the application of load. While the elastic behavior of the concrete is still perfect, the reinforced portion has necessarily a much higher coefficient of elasticity than the remaining portion, if the concrete is homogeneous. The neutral axis will hence be found between the reinforcing members and the middle of the section. But with the increase of the loads the elasticity of the concrete in tension changes more and more, while that of the concrete in

TABLE IV.

Number. 1	Materials. Metal. 2	Materials. Concrete: proportion of cement per cubic yard. 3	Elastic Limit of Metal. 4	Strength of Concrete. Tension. 5	Strength of Concrete. Comp'sion. 6	Percentage of Metal. 7	Ratio $K = \frac{E\ Conc.}{E\ Metal}$ 8	Distance from Neutral Axis to Tension Face. 9	Actual Compression of Concrete. 10	Moment of Resistance. 11	Cost per Cubic Yard. 12	Cost per Cub. Yd. per Foot-Pound. 13
		Pounds.	Pounds per Square Inch.					Expressed in h.	Lbs. p.sq.in	Ft.-lbs.	Dollars	Cts.
1	Iron	500	22,800	171	2,140	1	0.07	0.57	1,525	18.6	10.30	55
2	Iron	500	22,800	171	2,140	2	0.065	0.49	2,040	31.1	13.60	44
3	Iron	500	22,800	171	2,140	3	0.06	0.42	2,640	42.6	16.90	40
4	Iron	1,340.....................	22,800	427	5,120	1	0.10	0.57	2,390	25.6	15.80	62
5	Iron	1,340	22,800	427	5,120	3	0.09	0.46	3,260	49.4	22.40	45
6	Iron	1,340	22,800	427	5,120	4	0.087	0.42	3,830	61.0	25.70	42
7	Steel........	1,340	42,700	427	5,120	1	0.10	0.60	3,560	38.8	16.00	43
8	Steel........	1,340	42,700	427	5,120	2	0.085	0.51	4,560	61.7	20.00	33

compression changes at first only slightly. The neutral axis must thus shift, and it really does shift, away from the reinforcing members. Thus in tests made on numerous beams reinforcel by 1.1 per cent. of metal, the neutral axis, which was at first very near the middle of the section, has shifted up to 0.58 or 0.60 of the depth of the section with the increase in load. These figures are very near the value of 0.57 which is given in Table IV for 1 per cent. of metal.

A precisely opposite effect will take place at the end of the loading if the reinforcing members have too much area as compared to the concrete. This is because in order to balance the tension of the reinforcing members the concrete in compression must be very highly stressed, and the change in its elasticity will finally be the deciding element. The author has never experimented on beams so highly reinforced, and he is, therefore, not in a position to verify the accuracy of his conclusions as far as these beams are concerned.

7. The Influence of the Proportion of Iron or Steel.

A study of columns 10 and 11 of Table IV leads to the following conclusions: For the beams made of a concrete poor in cement, reinforced by ordinary iron, the moment of resistance increases from 18.6 to 42.6 foot-pounds when the percentage of the iron is increased from 1 to 3. The cost due to the increase in volume of metal grows much less rapidly than the increase in resistance. It would thus appear that there is a considerable advantage in increasing indefinitely the percentage of the iron if the figures of column 10, which give the stress of the concrete in compression, are ignored. To fully appreciate the meaning of column 10 some preliminary remarks are necessary.

A concrete containing 500 pounds of cement to the

cubic yard of sand and stone sustains a pressure of 2,140 pounds per square inch without alteration, and even much more when it is well made, but if the stress be repeated the concrete will crush under a much smaller load.

There is very little information on the effect produced on mortars by repeated loads. M. de Joly published in the "Annales des Ponts et Chaussées" a study which tends to prove that mortars in tension finally break under repeated stress at about half the tension which they can resist at a single application. Since concrete resists compression much better than tension, it is to be expected that this superiority in compression would show its good effects on the resistance under repeated compressive stresses, but in the absence of conclusive experiments it is prudent to admit that the repeating stress must in all cases remain below two-thirds of the crushing strength, which has been indicated in column 6 of Table IV. This is in accordance with the proportion determined by Woehler for metals. No similar progressive change in the nature of the reinforcing metal need be feared, for metal stressed within the elastic limit, because the experiments of Woehler have also proved that iron and steel can sustain an indefinite number of repetitions as long as the stress remains within the elastic limit.

If, therefore, it be assumed that the concrete will crush under the pressures given in column 6, or under two-thirds of their value, according to whether the load is permanent or repeating, it will be seen from an inspection of the table that a certain percentage of the metal must not be exceeded in order that the concrete shall not fail by crushing before the reinforcing metal has reached its elastic limit. By interpolation this limiting percentage is found at 2.17 per cent. for beams subjected to a permanent load, and at 0.8 per cent. for beams subjected to repeated application of loads. Among existing structures a great number can be found where the pro-

portion of iron reaches 2 and even 3 per cent. This is mostly due to the fact that in the experiments on which the design of the beams were based one or only a few applications of load have been made. The author believes that experiments with repeated application of loads will lead to somewhat different rules.

8. The Influence of the Quality of the Concrete and the Metal.

The knowledge that the danger of the crushing of the concrete under repeated application of load is nearer than generally supposed leads to the question whether larger proportions of cement could not be employed to advantage. The fourth to sixth lines of Table IV are based on a concrete supposed to contain 1,340 pounds of cement to the cubic yard of gravel and sand, mixed so as to give the highest resistance. By interpolation in column 10 it is found that such a concrete can sustain a proportion of metal of 5.6 per cent., if the load be constant, and about 3.3 per cent. with a recurring load. It thus may be concluded that the percentage of reinforcing metal can and should increase with the increase in strength of the concrete.

The above table also throws some light on the effects of a substitution of high steel, like rail steel, for the ordinary structural variety. For good and sufficient reasons such high steel is excluded from ordinary structures, and especially from riveted construction. It becomes very brittle after being punched or after having undergone deformation in the cold state, but it maintains a more than sufficient ductility when used in thin bars which receive no cold hammering. Its elongation, measured on a length of about 4 inches, is in this case at least 16 per cent. It thus seems to be advantageous to utilize its great resistance in concrete-steel construction.

Using this high steel with a concrete rich in cement,

and allowing for the steel unit stresses exceeding those generally allowed for iron and soft steel within the elastic limits indicated in column 4 of Table IV, moments of resistance are obtained as high as 38.8 and 61.7 foot-pounds, according to whether the per cent. of metal is 1 or 2 (seventh and eighth line). The choice between an increase in the proportion of the steel and the substitution of a steel of higher resistance, stressed almost twice the amount allowed for iron, depends on numerous considerations. An increase in the proportion of the metal results in increased rigidity, and the use of a steel working under the conditions indicated in the above, gives, on the contrary, to the reinforced members, a higher elasticity and ability to sustain without cracking twice as great deformations.

It will, therefore, be advantageous to use iron or soft steel for structures where the vibrations might prove dangerous, and high steel where spreading or unequal settling of supports are to be feared. Such, for instance, is the case when rigid abutments connecting various parts of a complex structure may, by unequal yielding, cause dangerous reactions among the connected members of the structure. The advantage of increased elasticity is manifest in this case. So it is also for structures liable to shocks. In fact, it may be stated that for an equal moment of resistance, a beam will absorb twice the kinetic energy when reinforced by a high steel as when reinforced by iron or soft steel because it will be in a condition to sustain double the deformation before breaking.

9. The Cost of the Different Types of Beams.

When the choice of reinforcing metal and of the proportion of concrete is not determined by purely engineering considerations it will be made in accordance with considerations of the greatest economy. To throw some light on the economic side of the subject, the cost of each kind of beam per cubic yard has been given in column 12 of

Table IV, and in column 13 the ratio of the cost to the corresponding moment of resistance has been indicated. A concrete containing 500 pounds of cement per cubic yard of sand and gravel can be easily had at $7 per cubic yard, all work of making the concrete-steel beams included. This is a good average price. An addition of 840 pounds of cement, per cubic yard of sand and gravel, will increase the cost by about another $7, but, allowing for the increased volume, the cost of the rich concrete used in Table IV will not exceed $12.50. The cost of wrought iron and soft steel has been estimated at 2.5 cents per pound, and that of the high steel at 3 cents.

Column 13 shows that the use of a rich concrete and a high steel will result in considerable economy. But to be more definite on the advantages offered by the latter variety of reinforced concrete, the ultimate crushing resistance of the concrete, which must not be exceeded, needs also to be considered. This consideration fixes the limits for the increase in the proportion of the metal.

If only the figures of the last column of Table IV are considered it must be concluded that the use of a high steel and a concrete rich in cement will result in economy in almost every case, and especially for beams subjected to repeated applications of load. But such an absolute statement cannot be made safely. Reasons for limitations exist which must be taken into account and which will change somewhat the aspect of the question. When using selected materials the least dimensions must be given to the beams, and if this should lead to a reduction in depth as well as in width a decrease in economy will be the result, because the moment of resistance is proportional to the square of the depth, while the cost is proportional to the first power.

It will be well, therefore, to limit the conclusion to the statement that a concrete rich in cement and reinforced by a high steel seems to be advantageous in certain cases, and that it should be studied, instead of being excluded

from concrete-steel structures, as has been the tendency heretofore. In any case, and engineers have already come to the understanding of it, a concrete rich in cement must be used for maritime works where impermeability is one of the main requirements for masonry. It also seems that liability to shocks and repetitions of load should, in general, lead to the use of good materials.

In preparing Table IV the author has taken the richest concrete and the strongest steel in order to emphasize the differences in the possible results, but it is probable that in many cases it will be better to use proportions of cement between those indicated in the table, and, it may be, steel of average strength, not so brittle as rail steel. There seems to be no objection to the substitution of medium and high steel for soft steel, reducing its volume only enough to make the cost the same; that is, about one-tenth. In fact, the elastic limit of medium steel exceeds that of soft steel by about 10 per cent., and if, therefore, the steel be used with this reduction in weight conditions will remain the same as if the soft steel were used, as long as the stresses in the reinforcing steel remain within the elastic limit. But when the elastic limit is exceeded, the beams reinforced by wrought iron or soft steel will crack and completely break to pieces, while the beams reinforced by the higher steel will not show any damage so long as there is no crushing on the compression side. This manifests an evident advantage which does not appear to be offset by any serious inconvenience.

10. The Determination of the Most Economic Proportion of Metal.

For a certain proportion of metal to concrete the compressive stress induced in the concrete reaches its limit, which cannot be exceeded without danger. If the proportion of metal be still increased, the stress caused in the concrete in compression cannot be kept within the

desired limits otherwise than by reducing the working tensile stress allowed on the metal. It is probable that the proportion for which the steel and the concrete in compression simultaneously reach their greatest allowable stresses will give the greatest possible economy. To determine the value of this critical percentage introduce into equation (1) the values of c and l, which will give the greatest allowable unit stresses, compute the value of x and introduce it into equation (2), which will furnish the desired proportion p of metal to concrete.

There will then remain unknown the moments of beams in which the proportion of steel is higher than found by the preceding method of computation. For the determination of these moments it is sufficient to establish formulas similar to (1), (2), and (3) by the introduction of a constant value for the compression c of the concrete, instead of the tensile stress l in the metal, which need not be considered, as it is known that the reinforcing has an excess of strength.

$$l = \frac{c}{K}\,\frac{x-u}{1-x} \qquad (4)$$

$$t\,x + \frac{c\,p}{K}\,\frac{x-u}{1-x} = \frac{c}{2}(l-x) \qquad (5)$$

$$M = e\,h^2\left(t\,x\frac{4-x}{6} + p\frac{c}{K}\,\frac{x-u}{1-x}\,\frac{x-3\,u+2}{3}\right) \qquad (6)$$

These formulas give the values of the breaking moments due to the crushing of the concrete for higher proportions of metal than the so-called critical percentage, while equations (1) to (3) give the breaking moments due to exceeding the elastic limit of the steel for the lower proportions of metal. In Table V are given the results as computed for beams made of the poorer concrete, 500 pounds of cement to the cubic yard of sand and gravel, and having iron for reinforcing. Both constant and repeated loads have been computed.

TABLE V.

Concrete with 500 Pounds Cement to the Cubic Yard, and Iron Reinforcing.

	Constant Loads.					Repeated Loads.		
Percentage of metal..........	0.82	1	2	*2.17*	3	*0.82*	1.5	3
Breaking moments, ft-lbs....	16.5	18.6	31.1	*33.2*	42.6	*16.5*	19.3	23.4
Cost per foot-pound, cents.	59	55	44	*43*	40	*59*	62	72
Tension in metal, lbs. per sq. in..	22,800	22,800	22,800	*22,800*	18,350	*22,800*	16,900	11,700
Compression in concrete, lbs. per sq. inch....	1,425	1,525	2,040	*2,140*	2,140	*2,140*	1,425	1,425

While the meaning of the figures of Table V is obvious enough, a diagram, Fig. 6, has been drawn for the con-

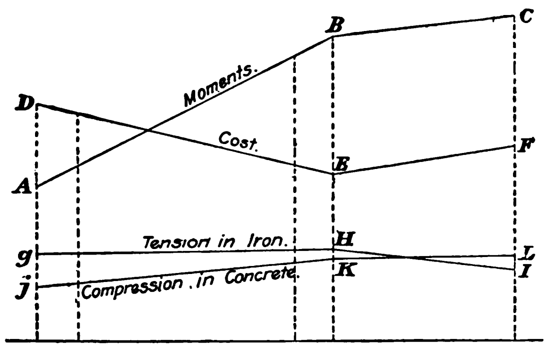

FIG. 6.

ı, with proportions of metal to concrete as
As ordinates for the curve A B C, the break-
nts were taken, for D E F the cost of a cubic
am per foot-pound sustained, for g H I, the ten-
square inch in the reinforcing iron, and for

j K L, the greatest compressive stresses in the concrete. The balancing proportion is 2.17 per cent. and evidently it is also quite advantageous as to cost per foot-pound, provided that beams of the same depth of cross-section are compared. It can be seen from the diagram that the cost increases less rapidly for an excess than for a deficiency of metal. The increase in cost in the first case is very slight if the excess of steel be kept within certain limits; and it seems that there is no great objection to an increase in the percentage of the metal, but its possible dangerous action should be kept in mind. In fact, with the reinforcing members relatively too strong, the concrete will fail first by crushing without preliminary signs of failure, while, if the reinforcing is too weak it will before failing first exceed the elastic limit and herald the approaching danger to the structure by the cracks which will develop in the concrete. Serious accidents could thus be prevented.

Tables VI and VII give the costs per cubic yard of beam per foot-pound sustained, which correspond to the different proportions of metal for beams rich in cement and reinforced by iron or steel. It is unnecessary to indicate here the tensile and compressive stresses since they vary according to the same laws as those given in Table V.

TABLE VI.

Concrete with 1,340 Pounds Cement to the Cubic Yard and Iron Reinforcing.

	Constant Loads.					Repeated Loads.			
Percentage of metal......	1	3	4	*5.6*	6.5	1	3	*3.3*	5.6
Breaking moments, ft.-lbs.	25.6	49.4	61.0	*84.5*	91.0	23.6	49.4	*53.3*	59.4
Cost per ft.-lb., cents.....	62	45	42	*37*	38	62	45	*44*	52

TABLE VII.

Concrete with 1,340 Pounds Cement to the Cubic Yard and Steel Reinforcing.

	Constant Loads.				Repeated Loads.		
Percentage of metal	1	2	2.5	3 5	1	1.2	2
Breaking moments, ft.-lbs	38.8	61 7	76.9	86.0	38.8	42.6	48.9
Cost per ft.-lb , cent	43	33	29	31	43	40	42

From Table VI it is seen that the theoretically most advantageous percentage of iron for beams, of a concrete as rich in cement as 1,340 pounds, and subjected to a constant load is 5.6. Such high percentages are practically out of question. It thus appears that a concrete containing about 850 pounds of Portland cement to the cubic yard will in most cases be sufficient for beams, reinforced by ordinary wrought iron or soft steel if they are to sustain largely constant loads. A richer concrete, on the contrary, will be advantageous for beams liable to repeated loads and reinforced by a steel of high resistance, the full capacity of which has to be utilized.

11. The Computation of Reinforced Concrete Beams.

Tables V, VI, and VII give for the various kinds of material and percentages of metal the bending moments which will cause either the cracking of the concrete in tension or the crushing of the concrete in compression, the limits to exceed which is dangerous. To compute the dimensions to be given the beams for practical purposes a suitable factor of safety must be adopted. It is well to consider first what has been done for iron and steel structures. For structures of some magnitude, where shocks are

little to be feared, ordinary wrought iron may be stressed up to 12,000 pounds per square inch, and soft steel to 15,000 pounds, or about one-quarter of the ultimate strength, and less than one-half of the least elastic limit. If it be considered that there is no metal structure in existence which would not collapse after exceeding the elastic limit, especially in its compression members, it must be admitted that the real factor of safety does not exceed two in the large structures. For small structures the greatest stresses are reduced to 9,000 and 12,000, respectively, and are sometimes still less; but this decrease is hardly sufficient to cover the increased effect of vibrations, shocks, and rust. These destructive causes need little to be feared for concrete-steel structures, and one would thus be tempted to adopt a factor of safety of 2 with respect to the breaking moments as calculated for the above tables and below which there is no danger of failure.

Before reaching a conclusion, however, the results obtained to the present time from reinforced concrete structures should be considered. These results are certainly too recent to furnish conclusive information as to the duration of such structures; many are, however, several years old, and the increasing application of concrete-steel structures for many purposes goes to show that they have so far proved satisfactory. It is, therefore, of interest to compare the breaking moments as computed in the tables to the bending moments which would be really applied to the beams according to the formula used by M. Hennebique.

Let M be the bending moment applied to a beam and $2H$ the depth of that portion of the section which is in compression. Assume that the compressive stress in the concrete is throughout the section equal to 350 pounds per square inch, and that its moment taken for the whole compressed portion must be in equilibrium with one-half of

M. If e denote the width of the beam, this moment of resistance due to compression in the concrete will be:

$$2\,He \times 350 \times \frac{H}{2} = 350e\,H^2 = \frac{M}{2}, \textit{ whence},$$

$$H = 0.038\sqrt{\frac{M}{e}}.$$

The other half of the moment M must be sustained by the metal, according to the Hennebique assumption. If then the distance of the metal from the line limiting the compressed portion be denoted by H, and if it be desired to stress its area A at 14,000 pounds per square inch,

$$A = \frac{M}{28{,}000\,H}.$$

It is unnecessary to prove that the Hennebique formula is based on two theoretical errors: First, the uniform distribution of the compressive stress over the whole compressed area, and, second, on the equality of the tensile and compressive moments with reference to the neutral axis.

The bending moments as computed by the above Hennebique formula are given in Table VIII in comparison with the breaking moments computed according to the author's formulas.

TABLE VIII.

Concrete with 500 Pounds Cement to the Cubic Yard and Iron Reinforcing.

	Constant Loads.					Repeated Loads.		
centage of metal.	0.82	1	2	2.17	3	0.82	1.5	3
aking moments in ft.-lbs.	16.5	18.6	31.1	33.2	43.6	16.5	19.3	23.4
rking moments (Hennebique)	7.2	8.3	11.8	12.3	13.5	7.2	10.0	13.5
tor of safety.	2.3	2.2	2.6	2.7	3.1	2.3	1.9	1.7

It can be seen that for structures subjected to constant loads, the factor of safety ranges from 2.3 to 3.1 and varies

little from an average value of 2.5. For an empirical formula this is a remarkable result and it shows the good practical sense of its originator. For structures subjected to frequently repeated loads, the factor of safety still remains satisfactory for small percentages in the neighborhood of 1 per cent., but it drops to 1.7 when the proportion of metal increases to 3 per cent., a somewhat too small a degree of safety.

From all the preceding considerations, we are led to the conclusion that as long as the condition of the oldest structures has not been fully investigated, it will be wise to adopt a factor of safety of 2.5 with respect to the breaking loads. These breaking loads will be determined for each type of construction after having made tests and having collected definite and reliable information on the materials to be used.

12. The Deformation of Concrete-Steel under Repeated Loads.

Until tests extending over a great length of time furnish exact information on the effects of repeated loads on concrete-steel, it will prove of interest to study the deformations which take place in beams immediately after the load applied to them has been removed; in other words, after subjecting the beam to a load, a repetition of which seems dangerous. For an example, test prism No. 35 has been selected, which had exactly the same composition as prism No. 34. This prism gave results almost identical with those of No. 34 during the period of the application of the load. In Table IX are the elongations and shortenings caused in the opposite faces of the prism when subjected to the bending moments indicated in the first line.

TABLE IX.

Bending moments in foot-pounds.	4.12	37.5	83.0	128.9	174.0	128.9
Elongations of concrete in thousandths of length	0	0.035	0.089	0.149	0.234	0.208
Shortenings of concrete in thousandths of length	0	0.008	0.057	0.125	0.213	0.200
Bending moments in foot-pounds.	83.0	4.12	219.7	280.3	219.7	174.0
Elongations of concrete	0.143	0.022	0.408	0.694	0.597	0.478
Shortenings of concrete	0.146	0.026	0.322	0.489	0.447	0.246
Bending moments in foot-pounds.	98.1	4.12	174.0	280.3	372.0	280.3
Elongations of concrete	0.313	0.084	0.432	0.729	1.137	0.929
Shortenings of concrete	0.231	0.052	0.296	0.506	0.741	0.647
Bending moments in foot-pounds.	219.7	174.0	98.1	4.12	....	....
Elongations of concrete	0.751	0.575	0.386	0.130	....	....
Shortenings of concrete	0.520	0.377	0.276	0.073	....	....

The load was applied and gradually removed three times after having caused bending moments of 174.0, 280.3, and 372.0 foot-pounds.

We will discuss simply the results of the deformations when the bending moment after having been increased to 372 foot-pounds was reduced to 4.1 foot-pounds, the value required to keep in place the levers of the loading apparatus. By means of computations similar to those made in the first section of this chapter, Table X, has been prepared. The depth of the prism, 2.362 inches, was divided, in columns 2 and 3, proportionately to the observed elongation and shortening, into two parts, one in tension and the other in compression. In column 5 the elongations of the metal have been tabulated as computed from the elongation of the mortar, admitting the common assumption of the conservation of plane sections. By multiplying the figures in column 5 by the coefficients in column 6, column 7 of unit tensile stresses in the iron has been obtained. Multiplying the total tensile stresses by the distances from the axis of the reinforcing metal to the resultants of the compressive stresses, the moments of resistance of the metal were obtained.

Subtracting these partial moments from the total bending moments in column 1, the resisting moments of the

TABLE X.

Moments. 1	Distance from Neutral Axis. To Compression Face. 2	Distance from Neutral Axis. To Tension Face. 3	Elongations in Thousandths of the Length. Mortar. 4	Elongations in Thousandths of the Length. Iron. 5	Value of E for Iron. 6	Tension in Iron. Per Square Inch. 7	Tension in Iron. Total. 8	Lever Arm of this Tension. 9	Resisting Moments Produced by Iron. 10	Resisting Moments Produced by Mortar. 11	Area of Mortar in Tension. 12	Lever Arm of this Tension. 13	Products of Cols. 12 and 13. 14	Average Tension in Mortar, Cols. 11 and 14. 15
Inch-lbs.	Inches.	Inches.			Lbs.p.sq.in.	Pounds.	Pounds.	Inch.	Inch-lbs.	Inch-lbs.	Sq.In.	Inch.		Lbs. p. sq. in.
4,460	1.032	1.330	1.137	0.903	29,870,000	26,970	1,780	1.71	3,040	1,420	3.24	1 35	4.37	326
3,380	0.992	1 370	0.929	0.743	29,870,000	22,190	1,465	1.73	2,530	850	3.34	1.35	4.50	189
2,640	0.984	1.378	0.751	0.601	30,010,000	18,040	1,190	1.73	2,060	580	3.36	1.34	4.50	129
2,090	0.957	1.405	0.575	0.463	30,440,000	14,090	930	1.77	1,650	440	3.43	1.34	4.59	96
1,180	1.016	1.346	0.386	0.307	30,720,000	9,430	620	1.77	1,100	80	3 28	1.34	4.39	18
50	0.870	1.492	0.130	0.106	30,860,000	3,270	215	1.81	390	—340	3.64	1.33	4.84	—70

concrete in tension and the corresponding portions of the concrete in compression are found. These figures are tabulated in column 11, and cannot be far from the true amounts, because they are the logical outcome of the observed deformations, if the assumption of the conservation of plane sections is admitted. This common assumption is very nearly true for beams subjected to a simple bending moment without shear, and such was the case of prism No. 35, which was tested in the same manner as prism No. 34.

To deduce with the least possible error from column 11, the values taken successively by the tension in the different elongated "fibres" of the concrete during the unloading a delicate and somewhat uncertain analysis would have to be made, which may well be omitted here. However, some useful indications, though not exactly precise, will be obtained by the aid of the following method: In column 12 are the areas of the portions of the section in tension, the products of the width of the prism, 2.44 inches, by column 3. In column 13, the lever arms of the tension in the concrete are given, computed on the assumption that during unloading all the elongated "fibres" had at the same instant the same tension, as actually occurred in almost all of them during the loading, from the time when the elastic limit for tension was exceeded. This assumption has not only not been proved, but it is probable that it is not at all true. Columns 13 to 15 must, therefore, be regarded as much less exact than those of column 11. They have, however, been computed because they are the only ones which afford even an approximate idea of the changes in the coefficient of elasticity, without which the true character of the phenomena caused by the repeated application of loads could not be clearly understood.

Column 14 gives the products of columns 12 and 13, and by dividing the resisting moments of the concrete in

tension in column 11 by these products, the average value of the tension in the elongated "fibres" have been obtained, in accordance with the assumption the uncertainty of which has already been stated. These average tensions are given in column 15. If now the variations of the tensions, $\varDelta t$, in column 15, and the corresponding variations of the elongations $\varDelta a$, obtained by taking the differences of the figures in column 4, are divided out, the successive values of the ratio $\frac{\varDelta t}{\varDelta a}$ are obtained. This ratio may be called the *instantaneous coefficient of elasticity,* because it characterizes the elasticity of the concrete at a given state of its deformation and free of the effects of the preceding deformations. Table XI contains these instantaneous coefficients of the concrete in tension as computed for prism No. 35.

It has been stated above that the tensile stresses in the concrete as given in column 15 of Table X, cannot be considered as very exact. The same is true of the instantaneous coefficients of elasticity, derived from them, column 5, Table XI; and while they should not be credited

TABLE XI.

Elongations of Mortar, a. 1	Values of Δa. 2	Average Tensions of Mortar t. 3	Values of Δt. 4	Instant. Coeff. of Elasticity: $\frac{\Delta t}{\Delta a}$ 5
In thousandths.		Lbs. per Sq. In.		Lbs. per Sq. In.
1.137		326		
	0.208		137	659,000
0.929		189		
	0.178		60	337,000
0.751		129		
	0.176		33	188,000
0.575		96		
	0.189		78	413,000
0.386		18		
	0.256		88	344,000
0.130		−70		

with an accuracy not possessed, their variations show a wide enough range to make with certainty some conclu-

sions and to give others a high degree of probability. From the figures it can be stated with assurance that *when a beam is gradually unloaded after having been subjected to a high bending moment, the instantaneous coefficient of elasticity of the portion in tension has at the beginning a high value;* and it is probable that it may reach, at least, one-fourth or one-fifth of the perfect coefficient of elasticity of the concrete from which this beam is made. *If the unloading be continued, the instantaneous coefficient of elasticity then decreases very rapidly,* and it is probable that it becomes about one-tenth of the initial coefficient of elasticity of the concrete.

The experiment has not been continued far enough to show what would take place if after the bending moment has changed its sign, the part at first in tension should gradually become compressed. It is probable that the coefficient would finally assume a value very near the initial value of the coefficient of elasticity of concrete in compression. It would then follow that between the high value at the beginning of the unloading and the very high value under high compressive stress, the coefficient passes through a minimum which appears to be less than one-tenth of the normal coefficient. This phenomenon should not be surprising. To show its meaning, if not its full extent, it may be stated that as shown in all the experiments, concrete-steel beams after unloading have only a small fraction of their deformation left.

To be more definite, take prism No. 35 as an example. Under a bending moment of 4,460 inch-pounds the elongation of the concrete went up to 1.137 of the length; when the moment was reduced to 50 inch-pounds the elongation dropped to 0.130, a decrease of 1.007. If, during this return to equilibrum, the concrete had maintained its initial coefficient of elasticity of 3,470,000, its tensile stress, first positive then negative, would have undergone in its extreme portion an algebraic

variation of $3{,}470{,}000 \times 0.001007 = 3{,}494$ pounds per square inch, giving some 6,350 pounds for the total stress in the tension portion of the initial section, which is erroneous. Since this portion of the section showed, under the action of the greatest moment, a total tension of about 1,050 pounds (col. 12 × col. 15 of Table X), it must after unloading dispose of a compression of $6{,}350 - 1{,}050 = 5{,}300$ pounds. But this is evidently impossible, because this pressure will not be in equilibrum either with the external forces, which have practically no value for the least moment of 50 inch-pounds, or with the tensile stress in the reinforcing metal which is only 3,270 pounds per square inch or 215 pounds total, as given in Table X. The shortened "fibres" are much too far away from this group of opposite stresses to effect their impossible equilibrium. The considerable decrease in the value of the coefficient of elasticity of the concrete in tension during the unloading of reinforced beams thus appears to be incontestible.

Another fact, though of much less importance, should also be stated here. The reinforcing metal remains in tension in reinforced members which have been unloaded after having been subjected to great bending, but this is in quite a small degree. In prism No. 35, for instance, the reinforcing bars kept an enlongation of 0.013 of 1 per cent. of the length and a tension of 3,270 pounds per square inch after an elongation of 0.1137 per cent., and a stress of 26,970 pounds.

It is of interest to determine the effect of a test load of greater amount than the repeated working load. When at the beginning of the experiment prism No. 35 was subjected to a bending moment of from 50 to 2,090 inch-pounds, without having been previously subjected to a higher load, the elongation of the extreme portions of the concrete varied from 0.0022 to 0.0254 per cent., the resisting moment of the iron from 43 to 695 inch-pounds

and that of the mortar from 6 to 1,391 inch-pounds. The results were very different when the bending moment was varied between the same limits of 50 to 2,090 inch-pounds after the prism had been subjected to a moment of 4,460 inch-pounds. The elongation of the concrete then varied from 0.013 to 0.0575 per cent., the moment of the iron from 390 to 1,650 inch-pounds, and the moment of the concrete from — 340 to 440 inch-pounds. It is thus seen that the phenomenon is quite different in the two cases, and that the application of a preliminary test load has the effect of considerably increasing the absolute value and the variations of the elongations of the concrete, of almost doubling the variations of the stress in the iron and of reducing those in the concrete by one-half.

It is impossible for the present to say whether this modification is advantageous and whether the concrete will more or less change its state by undergoing great deformations, producing, however, small stresses because of the considerable decrease of its coefficient of elasticity; or whether it will undergo small deformations enduring, nevertheless, a more considerable stress with a smaller reduction of the coefficient of elasticity. In any case, the reinforcing metal produces, as far as the resistance of the concrete to repeated loads is concerned, an effect not less important than that produced on the elongation under a single application of load. Indeed, the reinforced concrete of prism No. 34 has resisted without breaking ^52 repetitions of an elongation exceeding 0.127 per hile concrete not reinforced breaks under a repeti- ry slight tensile stresses, causing very small s. From experiments, the interesting results Joly has published in the "Annales des Ponts s," it is seen that prisms of neat cement broke tain number of repetitions of stress which, o the given coefficients of elasticity, correspond

to elongations much below 0.005 per cent., that is, at least twenty-five times smaller than the elongation which prism 34 sustained for a great many repetitions without apparent alteration.

In conclusion, it may be stated that the resistance of reinforced concrete to repeated loads is due to a small extent to the permanent tension which the deformations cause in the reinforcing metal and for the greatest part to a considerable decrease in the coefficient of elasticity of the concrete without a corresponding decrease in the tensile resistance. Deformations of concrete in reinforced members give it new properties very desirable for its resistance.

If bending moments be successively applied and removed alternately a certain number of times, the elongation of the portion of a reinforced beam in tension increases with each repetition, but by smaller and smaller amounts. Thus, on applying and removing four times the moment of 4,460 inch-pounds, prism No. 35 showed successive increases in elongation of 0.0023, 0.0019, 0.0015, and 0.0014 per cent. After these results it is almost certain that the successive deformations of reinforced beams decrease without interruption and tend toward zero when the repeated stress is kept within the limits in which an indefinite repetition of stress can be sustained. Experience and experiment will determine these limits. The progressive increase in the elongation of the concrete in tension necessarily causes an increase in the elongation of the metal embedded in it and, consequently, an increase in the stress of the latter. Observations which have been made and the preceding considerations seem to justify the following explanation.

When a reinforced concrete beam is subjected to repetitions of stress which do not exceed its resistance, its tension side elongates until, owing to the progressively increasing aid of the metal embedded in it, the tension

in the concrete is so reduced as to be within the limits of the stress, of which it can sustain an indefinite number of repetitions. After having worked under these conditions for some time the concrete, which has, so to say, economized its resistance, will doubtless regain its strength to its full extent if it be afterward subjected to higher stresses and deformations. Such, at least, was the case with prism No. 34, because the small pieces cut out of its tension side after 139,052 repetitions of a considerable elongation showed the same bending resistance as the identical concrete not subjected to fatigue. Only numerous and prolonged experiments can furnish data for the complete knowledge of the laws of resistance of reinforced concrete under repeated loads; but it seems quite probable that these laws are similar to the laws established by Woehler for metals, that is, that the limits of stress within which reinforced concrete can sustain an indefinite repetition are the higher the smaller the range of the stresses. If this hold true, the stresses must be reduced to a minimum when they change signs, and if, on the contrary, they are of the same character they can be raised to so much higher limits, the less the range of change below their maximum. For the sake of greater accuracy, the limits of the elongation of the concrete rather than the limits of stress should be spoken of, because the concrete can evade an excess of stress put on it by yielding sufficiently to throw the excess of stress on to the reinforcing metal. It is hardly necessary to remark that a higher proportion of cement while increasing the quality of the concrete must also raise the limits of stress or elongation within which the concrete can sustain any number of repetitions.

In the preceding, only one of the constituent elements of concrete-steel construction has been considered, namely, the concrete in tension. As to the reinforcing metal it is only necessary to remember that when embedded in

concrete it must never be stressed above its elastic limit, and the well-known investigations of Woehler have proved that under these conditions it does not undergo any alterations when subjected to an infinite number of repetitions of stress of the same character. As to the concrete in compression, the little that is known about it has been stated, and it is probable that the limiting stress for repeated applications of load is below two-thirds of the crushing resistance.

It is of a great interest to determine the elongations of the concrete, as commonly made, in beams of large dimensions, used in experiments or tested in existing structures. The author could not obtain any information from recently-made tests. The attention of the experimenters was not directed to this point, and to his inquiries it was replied that no arrangements were made for the observation of the beginning of the cracks and the corresponding deformations. But it is the current opinion among builders of concrete-steel structures that, as has already been stated, the concrete does not generally crack until the reinforcing metal is stressed nearly to its elastic limit, which requires an elongation in the neighborhood of 0.08 to 0.1 per cent.

13. The Assumption of an Increase in the Coefficient of Elasticity of the Steel.

All engineers who have studied concrete-steel construction agree, we believe, that the deformations of reinforced beams are so small, within their working limits, that they cannot be sufficiently explained by the assumption that the reinforcing metal alone takes care of the tension in the beam without any aid from the concrete. Convinced that reinforced concrete breaks under small elongations, as had been shown by tension tests of concrete not reinforced, several authors have assumed that

the properties of the iron or steel itself are modified by the adhesion of the concrete which surrounds it, and that its coefficient of elasticity is thus increased. This assumption should have been examined as to its correctness, but, after having consulted several mathematicians who are most conversant with molecular mechanics, the author considers it to be hardly admissible that of two combined materials the stronger should have its properties thoroughly modified by the relatively weak adhesion of the other on its surface. He believes the case is decided in the negative by the fact that the amounts of the bending moments sustained by the reinforced prism are explained by admitting that the concrete maintains its resistance well above the elongations which cause its rupture in members not reinforced, and also by the other fact that when cut off from the reinforcing the concrete, which underwent such an elongation, had maintained the necessary resistance to produce the bending resistance attributed to it.

It is hardly necessary to state that the resistance of the concrete in tension in reinforced concrete beams varies considerably with the proportions of the mortar, the proportion of the water used and the efficiency of the tamping. The tension reached about 300 pounds per square inch in all the prisms of series No. 34, which were made with exceptional care, but this is a maximum for mortars of ordinary proportions. In a series of ten experiments on concretes made without special care and of the proportions employed by M. Hennebique, the tensile resist- ᵔf the concrete fell to 170 pounds, and even to the ᵔf 115 pounds. It should, however, be stated mination, after rupture, of the prism which ow resistance showed that it had been made and that between the reinforcing bars the d received altogether insufficient tamping and ins of sand hardly in contact.

14. The Possible Results of Poor Workmanship.

The question naturally arises whether, even recognizing the property of concrete to sustain without breaking the elongations caused in reinforced beams, its tensile resistance should not be assumed as zero in computations for dimensions of proposed structures. It is well known that workmen are often careless and that poor workmanship of all kinds may occur during construction. It is especially certain that a lack of adhesion between different layers must always be expected, and that transverse cracks may result in reinforced members when the injured surfaces are perpendicular to the reinforcing metal, as is the case in vertical members. In horizontal beams, on the contrary, the layers are parallel to the direction of the reinforcing bars and it is more difficult to see how dangerous cracks may be developed because of faulty construction.

Observations on structures have proved that cracks perceptible to the naked eye are very rarely met. From the practical point of view it must be admitted that if there exist cracks, and that if, in spite of the effects of time, they remain such as never to be noticed, they are inoffensive as far as resistance proper is concerned. Like observations on existing structures, experiments prove that cracks do not indicate a near danger when they become visible, because the load must be much increased after their first appearance to cause noticeable deformations and final rupture.

These facts are reassuring as to the strength of concrete-steel constructions, and allay much of the fear of results of possible poor workmanship. But it is important to discover the causes of poor workmanship and to determine whether the factor of safety is the same for all types of construction. For this purpose compute the breaking moments which the different types of beams could sustain if the portion of the concrete in tension had

transverse cracks for the full width, and compare them to the breaking moments of uncracked beams as given in the preceding tables. To obtain values for the former, the expressions for the tension of the concrete, t, must be eliminated from both groups of formulas, (1) to (3) and (4) to (6). Results of such computations are given in Table XII, the first five columns referring to beams of a concrete not rich in cement and reinforced by wrought iron.

TABLE XII.

	Concrete with 500 Pounds Cement to the Cubic Yard, and Iron.					Concrete with 1,340 Pounds Cement to the Cubic Yard, and Steel				
Percentage of metal	1	2	2.17	2.4	3	1	2	2.5	3.3	3.5
Moment for beam without cracks. in ft.-lbs	18.6	31.1	33.2	34.4	42.6	38.8	61.7	76.9	83.0	86.0
Moment for beams with cracks, in ft.-lbs...........	14.3	27.2	29.6	32.4	42.6	27.2	51.9	63.8	81.2	86.0
Loss of resistance due to cracks, per cent........	23	12	11	6	0	30	16	17	2	0

A study of this table, together with Tables VI, VII and VIII, will show that the most economical proportion of metal is higher for beams with cracks. This was to be expected, because in these beams the reinforcing metal gets no aid from the concrete in tension to make up the moment required for the equilibrium of the concrete in compression, and it, consequently, requires more metal. For the beams with cracks 2.4 per cent. of metal is the most advantageous, giving a breaking moment which differs by not more than 6 per cent. from that of the beams without cracks, made of the same concrete and iron. If the percentage of metal be increased to 3 per cent., the breaking moments of the cracked and uncracked beams become the same. The appearance of cracks, as far as the resistance of the beam under a single applica-

tion of load is concerned, can, therefore, be considered as being of no importance for beams well reinforced.

This is easily explained. It has been shown that for the percentages of metal exceeding the most economical percentage the breaking moment must be computed in such a manner that the extreme portion in compression should not be stressed beyond the limiting compressive stress, thus getting the elongation corresponding to that indicated in Fig. 7 by the ordinate a A. The depth of

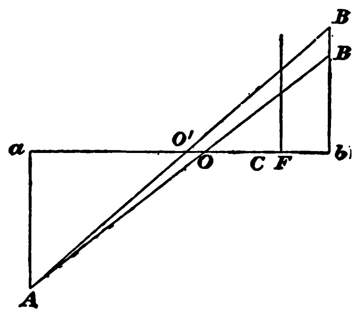

FIG. 7.

the cross-section of the beam is represented by a b. By solving equation (5) it is found that in beams without cracks the neutral axis cuts the line ab in a point, O, O b representing 0.42 of a b. Of course, the sum of the compressive stresses in the concrete in compression a O, on one side, and the sum of the tensile stresses in the concrete in tension O b and in the reinforcing metal F, on the other side, are equal.

The computation also shows that if the beam is cracked the neutral axis occupies a position O′ so that O′ b is equal to 0.44 of a b. The sum of the compressive stresses corresponding to the area of the new triangle O′ a A is somewhat smaller than the sum of the stresses corresponding to the initial triangle O a A; if the distance between the resultant of the compressive and of the tensile stresses is

increased sufficiently, the bending moment remains the same in spite of the cracks. In prisms with cracks the lever arm is the distance between the center of the metal and the center of gravity of the triangle O′ a A. In prisms without cracks it is only the tensile stress taken up by the metal which acts at F, the remaining stress being taken up by the concrete in tension with resultant acting at C, near the middle of O b. The average lever arm of the components of the bending moment is, therefore, appreciably greater in a beam with cracks than in one without.

Quite different is the result if the percentage of reinforcing metal is below the balancing or best percentage. As the percentage falls from 2 to 1 the loss in resistance caused by the cracks increases from 12 to 23 per cent. Similar results are found for beams made of a concrete very rich in cement and reinforced by high steel, to which the last five columns of Table XII refer. If, instead of employing the algebraic formulas which are approximate only, the exact curve of deformation were used, the figures obtained would not be perfectly identical; but the above conclusions would not be modified.

The conclusion is thus reached that the cracks in the concrete in tension materially decrease the resistance of beams reinforced by a low percentage of metal, but exert almost no influence on the resistance of beams in which the percentage of metal is appreciably higher than that giving the most economic beam. The decrease in resistance caused by the cracks does not exceed 11 to 17 per cent. if the most economic percentage is chosen. This offers a considerable guarantee for safety, at least so far as structures subjected to permanent loads are concerned. Experience only will teach whether cracks are also of so little danger for beams subjected to repeated loads, and whether the disintegration will extend more and more.

Another consequence of the preceding must be noted

here. When in reinforced beams the total breaking of the concrete in tension cannot cause a sensible decrease in their resistance, it seems that light cracks produced by any cause whatever cannot extend as long as the elastic limit of the metal has not been reached. The author has a small prism cut from prism No. 34 at the place where a slight crack was observed after the application of a moment of 6,830 inch-pounds. This crack did not extend any farther after 139,052 repetitions of stress. This result may serve as a good indication as to the effect of repeated loads.

15. The Influence of the Variations of the Coefficient of Elasticity of Concrete on the Resistance of Reinforced Beams.

It is well known that the properties, and especially the coefficient of elasticity, of mortar and concrete vary with the quantity of water used and the efficiency and duration of the tamping. It is important to determine the consequences which result from this in concrete-steel construction. For beams of a concrete containing a small proportion of cement, reinforced by 1 per cent. of metal, for example, the ratio of the coefficient of elasticity of the concrete to that of the steel, K, can in practice reach the value of 0.11 instead of 0.07, as was assumed for it in Table IV. By adopting this value it is found that the breaking moment will then increase to 22.4 instead of 18.6 foot-pounds. The variation will thus not be more than 17 per cent. of the resistance, while the value of the coefficient of elasticity has increased 60 per cent.

Similar results are obtained for beams strongly reinforced. Thus, for beams having 3 per cent. of metal, an increase in the value of the coefficient of elasticity of 60 per cent. causes an increase in the breaking moment of less than 17 per cent. Decreases in the value of K,

of course, cause similar effects in the opposite direction. The influence of the variations of the coefficient of elasticity of the concrete on the resistance of reinforced beams is thus quite slight, and this fact is explained by the displacement of the neutral axis.

Reinforced concrete possesses thus the two following properties of great practical importance. When the proportion of reinforcing metal is sufficient, cracks exert a slight influence on its resistance to bending and they show no tendency to grow. Again, the coefficient of elasticity of the concrete can vary within wide limits without causing a proportional change in the resistance of the reinforced beams. This change is somewhat below one-third of the variation of the coefficient of elasticity. Thus, as far as the bending resistance is concerned, which is the only resistance so far considered, the consequences of poor workmanship are less dangerous than might have been expected. As has been indicated before, a factor of safety of 2.5 will be sufficient.

16. Symmetrical Reinforcing.

The idea of reinforcing concrete with iron was inspired by its low tensile resistance, which is, on the average, not more than 1/10 to 1/12 of its compressive resistance. Recently it has occurred to many engineers that it can be shown that the widely-extended practice of reinforcing beams assymmetrically, that is, placing the reinforcing metal in the tension side only, is not rational and that the highest resistance and economy will be obtained by symmetrical reinforcing. The reasoning is strictly correct in the logic of its deductions, but it assumes at the start that the coefficient of elasticity of concrete is the same for tension as for compression; and that this holds true not only for the very small deformations, for which this assumption is correct, but also for the much greater

elongations which take place in reinforced beams, and even up to the ultimate strength, which is of very great importance, because of the elimination of a given quantity expressed by the factor of safety. Not only does the coefficient of elasticity of concrete decrease considerably as soon as the elongation exceeds 0.01 to 0.015 of 1 per cent., but it soon becomes almost zero; and the tensile resistance remains constant, while that of the concrete in compression continues to increase rapidly. It is, therefore, quite difficult to admit that it is of advantage to reinforce concrete beams symmetrically where they are not subjected to reversals of bending moments, which is the only kind of beams treated here. However, if it is not rational to reinforce symmetrically a material the properties of which are essentially unsymmetrical, it does not follow that there is never any advantage in reinforcing the compressed side of concrete beams by a less section of metal than that used for the tension side.

For example, let us find the result which will be obtained in reinforcing in this manner the prism reinforced by 2 per cent. of metal, to which the second line of Table IV refers. If two symmetrical reinforcing sets, each having a section equal to one-hundredth of the cross-sectional area of the beam, be added to its previous reinforcing, the breaking moment will be increased by 14.4 foot-pounds, and will thus become 45.5 foot-pounds, an increase of about 46 per cent. The cost per cubic yard will be increased from $13.60 to $22.20, or by about 48 per cent. This would give a beam reinforced in its tension side by 3 per cent. and in its compression side by 1 per cent. of metal. Referring to the different tables, it will be seen that better results can be obtained at less cost by increasing the proportion of cement at the same time with the percentage of the reinforcing metal.

17. Conclusions.

We have observed how the quality of concrete can vary with the care given to its fabrication, and, therefore, understand that even in experiments made at laboratories for mortars considered identical, values of the coefficient of elasticity have been found varying from 1,400,000 to 5,700,000 pounds, and even more. In the author's first experiments, where the making of the test specimens was not sufficiently watched, he found coefficients which approached the one or the other of these extreme values. It seems that such variations must also be met in practice. With such widely differing results it is impossible to give figures which should hold true for all cases, and the author, therefore, does not attribute an absolute value to any of the figures given in this treatise, except, of course, those resulting directly from each experiment, in so far as they apply to the prisms for which they were obtained.

On the other side, the figures given in the different tables are deduced from algebraic formulas, which are not absolutely exact because in their deduction the two parts of the curve of deformation were replaced by straight lines. The error so committed is not appreciable except for the value of the compressive stress in the concrete, which is actually noticeably lower than the one given by the computation. The figures obtained in the above have no value if they are considered separately; but since they have all been deduced by the same method from identical or concurring data, their relations are of importance, and the laws which appear to result from these relations well deserve attention. They are not presented here, however, as being finally established; and it is desirable that numerous experiments be made to enable engineers to accurately establish the laws of behavior of reinforced concrete. It is with the above re-

strictions in mind that the following conclusions are drawn.

Concrete sufficiently reinforced by iron or steel can, without cracking or disintegrating, sustain elongations much higher than those observed in the ordinary tension tests. The reinforcing metal cannot insure the uniform elongation of the concrete when the elastic limit of the metal has been reached.

When the elongation of the reinforced concrete exceeds the ordinary elongation due to tension, the tensile stress increases more and more slowly and the coefficient of elasticity consequently decreases more and more rapidly. The tensile stress becomes almost constant, and the coefficient of elasticity is very low from the point where the elongation reaches the greatest value due to bending in unreinforced concrete, which is 2 to 2.5 times as much as the elongation due to direct tension.

When a beam which has sustained a high bending moment is gradually unloaded the coefficient of elasticity of the concrete in tension has at the beginning quite a high value, though much below the initial coefficient of elasticity; then it decreases to a very low value.

If concrete be subjected to the repetition of stresses below the greatest stress which it has once resisted, it sustains so much the greater deformations and undergoes so much the smaller changes, the more the greatest stress has exceeded the present stress.

The curves of deformation of tests, the results of which are known to the author, agree with these facts and the explanations given above. It thus appears to be logical to eliminate the hypothesis of an increase in the coefficient of the metal embedded in the concrete.

The curve of deformation of a concrete, plotted from the results of experiments, enables one to determine graphically all the stresses which are developed in reinforced beam made of the same concrete, when it is sub-

jected to simple bending moment. Algebraic equations easily furnish a sufficiently accurate solution.

For a given quality of concrete and metal it is thus easy to compute the most economical percentage of metal which is characterized by the fact that the metal and the concrete in compressing both reach their allowable limits at the same time.

The allowable limit of stress is doubtless for the concrete in compression, as well as for all materials which have been investigated, so much lower, the more frequently the stresses vary and the wider their range.

The typical percentage of reinforcement for a beam is the higher the higher the resistance of the concrete and the lower the coefficient of elasticity, and especially the elastic limit. For beams subjected to repeated loads, the percentage of the metal must be reduced or, what is better, the proportion of cement increased.

The substitution of steel of the quality of rail-steel for wrought iron, with a section reduced in inverse proportion to the cost, seems to have advantages only when the reinforcing metal will not undergo deformation before being put in place. The use of rail-steel stressed in proportion to its higher elastic limit is dangerous where stiffness is required, and of advantage where ductility and resistance to impact is required.

When the breaking moments have been computed the factor of safety must be determined. It appears that in accordance with the results obtained from existing structures the factor of safety can be fixed at 2.5.

The transverse rupture of the portion in tension decreases materially the resistance of weakly-reinforced beams, but it has little effect upon beams which have a reinforcing as high as the typical percentage. It has no influence at all on beams reinforced by a still higher percentage. The cracks thus have no tendency to extend when the reinforcing is sufficiently strong.

The variations of the coefficient of elasticity of the concrete exert only a relatively small influence on the resistance of reinforced beams. Because of these two facts, bad workmanship in the construction of concrete-steel beams is less to be feared than might have been supposed, as far as their resistance to simple bending moments, without shearing stresses, is concerned.

Symmetrical reinforcing is not to be recommended for beams which will surely always bend in one direction, because the coefficient of elasticity and the resistance of the concrete are essentially unsymmetrical in bending. Some advantages may be obtained by adding symmetrical reinforcing to the unsymmetrical reinforcing required to obtain equilibrium by balancing the difference in the resistance of concrete in tension and in bending; but it seems that the same result will be obtained, at less cost, by increasing the proportion of cement and the section of the reinforced metal in tension.

CHAPTER II.

The Deformation and Testing of Reinforced Concrete Beams.

1. The Necessity for Direct Tests of Reinforced Concrete Structures.

The engineer who is designing a steel structure specifies that tests shall be made at the shops which will give a clear indication of the character of the materials used. These tests refer to the ultimate strength, elastic limit, ultimate elongation and reduction of area. He also inspects the construction of his structure, and bad workmanship is getting more and more rare. If poor work is sometimes done it can be discovered by careful inspection, and when the structure is tested on completion nothing unexpected will take place, if its type and design are based on practical experience. Thus the deflections which steel structures show under their test loads are found to be almost identical with those computed for them and their determination is, therefore, not of great value.

The measuring and observation of the local deformations, on the contrary, furnish valuable information on the distribution of stresses, and enable the engineer to appreciate the advantages and disadvantages of the various types of construction; but it is very seldom that they disclose faulty construction or bad material. It can thus be said that for steel or iron structures the preliminary tests of the materials used and the inspection of construction and erection furnish all the necessary assurances. Quite different is the case with concrete-steel structures, because laboratory tests tell us only of the quality of the materials employed, and the most active inspection will not be able to prevent positively poor workmanship and

faulty construction, which can destroy the strength of structures made of the best materials.

In fact, the proportions of the concrete may, in spite of careful watching, not be in all parts in accordance with the specifications; the quantity of water used in mixing must, in order to produce identical results, vary within a wide range, according to the condition of moisture in the materials and the atmosphere, and it is quite sure that it will be sometimes badly proportioned. If too much water be added the strength of the concrete, and especially its coefficient of elasticity, will be decreased to a degree which may be considerable; if too little water be added the adhesion of the concrete to the reinforcing metal will not be sufficient. The thoroughness of the tamping has a still greater influence on the strength of the work. To the faults of execution, faults of design may be added. The latter must especially be guarded against in a new type of construction, the theory of which is not yet fully established.

2. The Insufficiency of the Assumptions Made for the Computation of Deformations.

Whatever the results of the tests of the materials may be, very little information on the strength of a concrete-steel structure can be obtained without direct tests of the structure itself. But observed deformations will furnish really useful indications only when compared to the normal deformations which, according to the computations, should have been caused in accordance with the quality and disposition of the materials. Hitherto no method has been established to enable the engineer to compute these normal deformations. However, at one time it was assumed that reinforced concrete, whatever its deformation, preserves the coefficient of elasticity as determined in ordinary tension tests, and at another that the resistance of

the concrete in tension can be neglected in reinforced members. Sometimes it has also been assumed that the resistance of concrete in tension can be neglected only when its deformations exceed the elongation which causes rupture in common tension tests. None of these assumptions has given results agreeing with the actual behavior of reinforced concrete. We may conclude that in reinforced concrete construction certainly some particular phenomena occur, a knowledge of which is necessary to predict their resistance and deformations.

3. The Law of Deformation of Concrete in Compression.

Concrete in compression is generally not reinforced and it cannot be expected that the phenomena mentioned in the preceding section will be found to be caused by it. It suffices to recall the well-known law of deformation of concrete in compression and to make it more precise by naming the ratio of an infinitely small variation of compression to the variation of length caused by it, the "instantaneous coefficient of elasticity."

When the compression increases, but remains within an amount which, in general, is nearly a third of the ultimate strength, the instantaneous coefficient of elasticity decreases, but in a very small degree. When the stress increases still more the change in the elasticity gradually increases; it is appreciable for a compressive stress near one-half the value of the ultimate, and it then increases so rapidly before failure that the instantaneous coefficient may fall below one-twentieth of its value under a light stress. This aspect of the deformation is similar to the one generally observed on all materials. The simple law which determines the deformation of concrete in compression thus cannot furnish the explanation of observed irregularities, and it must be looked for in the phenomena which are produced in tension.

4. The Law of Deformation of Concrete under a First Tension Load.

Engineers who have studied the subject have encountered the two apparently contradictory facts that mortar and concrete of the usual proportions break in tension with elongations below one-ten-thousandth of the length, and that in reinforced concrete beams which have sustained on their tension sides elongations up to one-thousandth of the measured length, no cracks are observed. It was, however, not known whether the portions which apparently are not injured do not contain invisible cracks which are prevented from becoming visible by the reinforcing metal, and whether they have not been altered in some other way, so as to reduce their resistance.

Of the experiments which the author undertook to investigate this subject, those on reinforced concrete prisms in bending have been described and discussed in Chapter I. Experiments on reinforced concrete prisms in simple tension were made afterward, and their results are given in Chapter III. It would require a long and intricate analysis to show how the laws of simple tension can be deduced from the complex phenomena of flexure. A few words will, on the contrary, suffice to explain the method followed to establish these laws by means of the results of simple tension tests.

Mortar prisms reinforced by symmetrical reinforcing members were subjected to tension acting in the line and direction of their gravity axis. For each load were measured, first, the elongations of the reinforcing members, and, second, those of the sides of the prism. The latter were found to be practically equal to the elongations of the reinforcing members, except at their ends. To an elongation λ, of the reinforcing members, having a section a and a coefficient of elasticity E_s, corresponds a stress in the metal equal to $\lambda \ a E_s$. If the total load on

the prism which has caused these deformations is denoted by P; the total stress in the mortar was $P - \lambda a E_s$ and its tensile stress per unit of area will be $\frac{P - \lambda a E_s}{A}$, A being the cross-sectional area of the mortar.

5. The Effect of Repeated Stresses.

Knowledge of the curve of deformation corresponding to a first load will be sufficient for computing the deformations for the usual tests of reinforced concrete structures, which include a single loading only; but it is important also to know how these structures will be deformed under repeated test loads. In Fig. 8 the curves

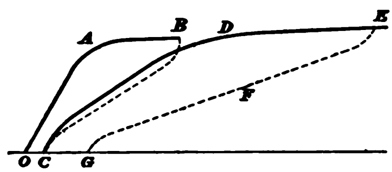

Fig. 8.

represent the law of deformation of the mortar in a reinforced mortar prism subjected to repeated and increasing loads, alternating with returns to equilibrium. If the load is gradually decreased after first having continued the test to the point B, the curve B C, which represents the deformation during the period of unloading, is almost a straight line except at its ends. The permanent deformation, which remains after complete unloading, is indicated by O C. If the prism is loaded again to E and again unloaded, the curves C D E and E F G will be successively obtained. These curves, like B C, are almost straight and their inclinations to the horizontal are so much smaller as their deformation has been extended further.

The precise notion of the instantaneous coefficient of elasticity; that is, of the inclination to the horizontal of the tangent to the curve of deformation, enables us to summarize briefly the laws of the deformation of mortar or concrete in tension in reinforced concrete members. These laws are as follows:

1. During the first loading the instantaneous coefficient of elasticity is equal to that of unreinforced mortar, as long as the elongation does not exceed the amount which will cause rupture in the same mortar when not reinforced, and it decreases very rapidly as soon as this amount is exceeded, approaching zero.

2. When a reinforced concrete member is subjected to a successive series of unloadings and reloadings the instantaneous coefficient of elasticity has, during any of these operations, an almost constant value, and this value is the smaller the greater the maximum deformation has been.

3. The repetition of the same stresses causes a decrease in the coefficient of elasticity which is at first appreciable, but approaches zero.

6. The Effect of Very Small Cracks on Stress and Deformation.

These laws inform us what a perfect mortar or concrete *can do* and not what will be certainly obtained from materials as they are used practically. Reinforced concrete members sometimes contain cracks, and this must be taken into account in computing their proper dimensions. It is an established fact that in well-built reinforced concrete structures, which are not exposed to an excessive drying out, only very seldom are cracks visible under a magnifying glass produced before or during the usual tests. Furthermore, laboratory experiments prove that the cracks, caused by sufficiently increasing the load above

the generally accepted test loads, have no appreciable influence on the deformation as long as they cannot be seen with the naked eye. This becomes evident when it is considered that a crack influences only a very small fraction of the length of the tested member. It must, therefore, be concluded that the deformations caused during the test loads generally used for structures of this type are not influenced to any appreciable degree by the invisible cracks which may have existed before the test or have been produced during the same.

7. The Exact Method for the Determination of the Deformations.

The law of deformation of the reinforcing metal and the concrete of a concrete-steel member gives all the elements necessary for computing the deformations under any load sufficiently moderate not to cause wide cracks, and especially under the test load. The comparison of the actual and computed deformations under the test loads will furnish valuable information not only on the resistance of the member tested, but also on the nature of the danger most likely to threaten it. This problem of computing the deformation for a given stress can be solved only by successive trials to find the position of the neutral axis, and such an inclination of the line A′ B′, of Fig. 4, that the resulting stresses will satisfy the double condition that the sums of the tensile and compressive stresses shall be equal, and that the moments produced by the stresses shall be equal, during the test.

8. The Approximate Method of Computation of the Deformation.

It is a waste of precision to attempt absolute accuracy in computations based on the physical properties of concrete.

It is, therefore, proper to substitute for the deformation curve Q P O M, in Fig. 4, the polygon R O L M, which is very little different, but allows the introduction of simple equations in the usual case of concrete beams, reinforced on the tension side only. In Chapter I, section 5, the polygon R O H M was taken, including the small tri-

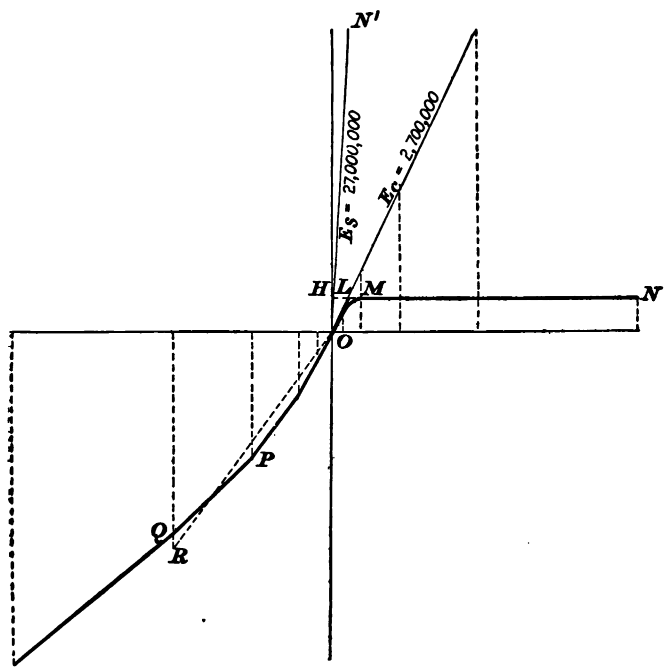

FIG. 4.

angle O H L, which was sufficiently accurate for the purposes there intended, but for the computation of the deformations the more accurate polygon R O L M will be used. Using the same notation as before and denoting by s the tensile stress in the reinforcing metal in pounds per square inch, the following equations are obtained:

From the assumption of the conservation of planes after flexure, it follows that the tensile stress s in the reinforcing metal is to the compressive stress in the extreme portions of the concrete c as the products of the distances from the neutral axis by the coefficients of elasticity of the two materials. Hence:

$$c = Ks \times \frac{1-x}{x-u} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

The condition that the sum of the tensile stresses in the metal and concrete must equal the sum of the compressive stresses in the concrete furnishes the following equation:

$$tx - \frac{t^2}{2\,c}(1-x) + sp = \frac{c}{2} 1 - x \dots\dots\dots\dots\dots\dots (8)$$

The moment of resistance of the reinforced beam which is produced by the tensile stresses in the reinforcing metal and concrete in tension and by the compressive stresses of the concrete in compression is:

$$M = e\,h^2 \left[\frac{t\,x^2}{6} - \frac{t^3}{6c^2}(1-x)^2 + sp\,(x-u) - \frac{c}{3}\,(1-x)^2\right] \dots (9)$$

The tests which will generally be made on the metal and concrete give the values of t and K; only three unknowns thus remain to be evaluated x, s, and c. The three equations (7), (8), and (9) furnish the means for determining their values corresponding to each value of the test moment M. The form of the equations is such that their direct solution will be difficult, and the same practical result can be obtained in the following manner. From equations (7) and (8) eliminate s, then:

$$t\,x - \frac{t^2}{2\,c}(1-x) + \frac{p\,c}{K}\,\frac{x-u}{1-x} = \frac{c}{2}\,(1-x) \dots\dots\dots\dots (10)$$

This equation contains three variables c, K, and x. If the value of the compression c in the concrete is taken arbitrarily, the curve of deformation of the concrete used in the structure will give the value K, which corresponds to this compression. Introducing the values of c and K in equation (10), the corresponding value of x will be found.

Substituting the numerical values of c, K, x thus found in equations (7) and (9) the corresponding values of s and M are obtained.

By giving c a certain number of successive values, conveniently chosen between the limits which may be caused under the test load, a series of values of c, s, and M will be obtained, each of which will satisfy the equations and will consequently characterize one of the possible positions of equilibrium of the beam in question. If the ultimate strength of the beam is to be determined, the values obtained are just the elements required; however, it is not the deformations which will be caused by s and c which are looked for, but the deformations in the beam under normal conditions when tested by the usual trial load.

The elongation of the metal λ is equal to $\frac{s}{E_s}$ and the shortening of the most compressed portion of the concrete r is equal to $\frac{c}{E_c}$ where E_s and E_s are the coefficients of elasticity of the metal and concrete. The coefficient of elasticity of steel is known; that of the concrete for each value of c is equal to the E_s multiplied by the value of K corresponding to that of c, previously determined for equation (10). To apply the results to all beams having the same proportion of metal p, whatever their width e, and their depth h, the total moments M must be reduced to the unit moments $m = \frac{M}{eh^2}$, that is, the resisting moments for a beam of unit width and depth.

The values M, s, c which will be obtained will enable the designer to compute m, λ, r, each of which will be realized in the tested beam when the test load produces the absolute moment M, and the moment m referred to a section a unit deep and a unit wide. By numerical interpolation the deformations λ, r, which will be caused under the action of any unit moment m, whose value lies

between the extreme values, can be easily determined. Still more easily will the same result be obtained by constructing two curves having for abscissas values of m and for ordinates the corresponding values of λ in one of them and r in the other. Such curves are shown in Figs. 9 and 10.

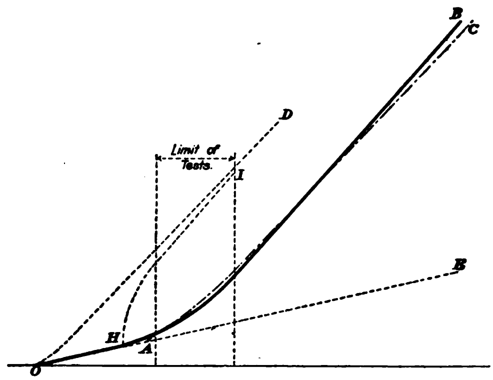

FIG. 9.—CURVE OF ELONGATION OF REINFORCING METAL.

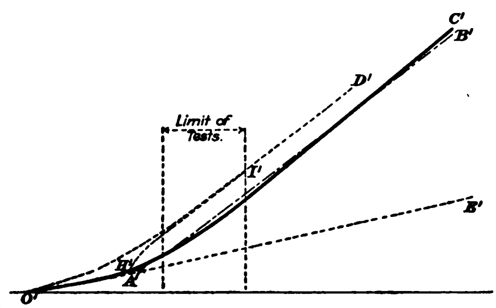

FIG. 10.—CURVE OF GREATEST SHORTENINGS OF CONCRETE.

The curves A C represent the elongations computed by formulas (7) to (10), and the lines O E, O D, O H I, the results of the three assumptions made by different writers, which will be considered below. The full line O B represents the actual deformations of prism No. 34.

9. The Complete Curves of Deformation.

If the assumption for the deduction of equations (7) to (10) be remembered, it is seen that these are not applicable before the elasticity in the extreme portion of the concrete in tension has changed its character. For loads causing stresses below the above assumption the coefficient of elasticity preserves its normal value, and it is very easy to compute the deformations by the usual formulas. But care must be taken in computing the moments of inertia to account for the difference in the initial coefficients of elasticity of the metal and concrete. Completing thus, for light loads, the curves given by equations (7) to (10) for the period when the elasticity of the concrete in tension is changed, the dot-and-dash lines O A C and O′ A′ C′, Figs. 9 and 10, are obtained. These lines will necessarily show up any irregularity, as R L M, in Fig. 4, which has served as a basis for the deduction of the formulas. It is evident that the real condition will be still more nearly approached in joining by curves the two sides of the angle formed by O A C, as the curve of deformation of the concrete in tension joins both lines R L and L M.

Finally the two complete curves O A C and O′A′ C′ will be obtained. The former represents the elongations of the reinforcing members, and the latter the greatest shortenings of the concrete which will be caused in a beam reinforced by the proportion of metal p when subjected to a unit bending moment; that is, to a total bending moment $M = m\ e\ h^2$, e being its width and h its depth. Similar curves may be constructed for a number of values of the proportion of metal p, and all necessary elements will then be at hand to determine at once, by numerical or graphical interpolation, the deformations of any beam of reinforced concrete which has the characteristic qualities E_s, K, and t for which the computations have been made.

10. The Comparison of the Actual Deformations with the Computed.

The method just given should be checked experimentally to determine its reliability. Prism No. 34 will again be taken for comparison. The deformations observed during the test are shown in full lines in Figs. 9 and 10, and the deformations computed by formulas (7) to (10) in dot-and-dash lines. In each diagram the two curves are almost identical, and it is natural to question whether this remarkable agreement does not result from the fact that the values of K and t introduced in the formulas have been determined by a method which should lead to this agreement. Such is not the case.

E_s was determined by a tension test on a wire identical with the reinforcing wires. The experiments gave the position of the neutral axis, and, consequently, for each load the area of the compressed portion of the prism, thus permitting the computation of the compressive stresses in the mortar within limits of accuracy which would still be very narrow if even appreciable errors were committed in the evaluation of the tensile stresses t in the concrete. These values of the compressive stresses once being known, the values of K were determined by the corresponding values of the shortenings of the concrete, which were measured directly. The tension t, maintained almost without variation from the time the elastic limit was exceeded, was determined by direct experiments made on small prisms detached from the reinforcing, as described.

Hence for prism 34 formulas (7) to (10) have served to compute the deformations with a very satisfactory accuracy by introducing in them the real characteristics of the mortar and iron employed in its construction.

11. Results of Assumptions Made by Different Authors.

To the author's knowledge only three assumptions have been made in attempting to explain the resistance and deformation of reinforced concrete beams. The first assumption, that the concrete preserves its coefficient of elasticity throughout the test, no matter how improbable it is, forms the necessary basis for certain methods adopted by some engineers, who assume that in the computations the reinforcing metal can be replaced by an equivalent area of concrete in the ratio of the initial coefficients of elasticity of the two materials. This assumption gives for the computed deformations the ordinates of the two straight lines O E, O′ E′ of Figs. 9 and 10. They differ much from the actual deformations, except under light loads. The second and directly opposite assumption, that the concrete in tension has no effect on the resistance, has been made up to the present time by the majority of practitioners, and the author has explained in Chapter I, how it happened that this assumption, though inaccurate, has given acceptable results, in the computation of dimensions of reinforced concrete beams. Quite different is the case of the deformations. The values on this assumption are obtained by means of equations (7) to (10), by making $t = O$. They are represented by the ordinates of the lines O D and O′ D′, Figs. 9 and 10.

The third assumption, apparently the most reasonable before the experiments described in Chapter I were made, that the concrete remains perfectly elastic while its elongation does not exceed the value causing rupture in simple tension tests of unreinforced concrete — which is true — and — which is not true — that it breaks as soon as this limit is exceeded, requires that in a beam subjected to an increasing bending moment cracks start in the extreme portions when the elongations become equal to breaking elongations, and as the load increases, that the cracks grow

progressively until approaching very near to the neutral axis. If the beam would actually behave so, the deformations would be represented during the elastic period by the ordinates of the lines O E and O′ E′; they would then rapidly increase, owing to the rupture of the portion in tension, the resistance of which would approach zero, and they would then be represented by the lines O H I D and O′ H′ I′ D′ forming asymptotes to the lines O D and O′ D′.

Though it is interesting from a theoretical point of view to see how the different assumptions agree with the facts observed until rupture, it is especially useful to study the differences shown within the limits of loads which may actually be used for testing purposes. These limits are drawn on the diagrams and the differences shown, within these limits, between the computed deformations and those observed are given in the following table. These figures lead to several conclusions:

TABLE XIII.

	Elongations of Iron.	Elongations of Concrete.
Difference between actual deformations and results of formulas (7) to (10), per cent.	5	5
Difference between actual deformations and results, first assumption, per cent.	45	50
Difference between actual deformations and results, second assumption, per cent.	110	25
Difference between actual deformations and results, third assumption, per cent.	100	20

It is seen at a glance that all the assumptions previously made have predicted deformations very different from the

actual, while a very satisfactory approximation is obtained by the use of formulas (7) to (10). Within practical limits, the third assumption gives almost as poor results as the second, owing to the fact that it is true only during the period of perfect elastic behavior, or for loads not exceeding the same, while practical tests require much heavier loads.

12. Effect of Percentage of Iron and Quality of Concrete.

The errors resulting from the second assumption are due to the behavior of the concrete in tension, and must, therefore, increase with the relative importance of the concrete to the reinforcing. Prism No. 34 was in conditions favorable to this. It should be noted that the percentage of reinforcing metal of 1.12 per cent. is smaller than the percentages generally adopted, and that, therefore, the relative importance of the mortar is greater, its importance being increased also by its quality. It was made very carefully and showed a tensile resistance of about 300 pounds per square inch, at least 30 per cent. higher than sustained by concretes generally employed in building construction. For ordinary structures which set in air, the second assumption must, for these reasons, lead to smaller errors than are shown in the comparison with the results on prism 34. On the other hand, if for hydraulic works rich concrete is used, a still higher resistance will be obtained for the concrete than that of the mortar of prism 34, and the difference between the results of the different formulas will be increased.

13. Effect of Longitudinal Sliding of the Reinforcing Metal.

The experiments on prism No. 34 differ from most practical tests, also in that the force causing the bending

moment was applied near the ends, and, hence, there was no shearing stress in the central portion where the deformations were observed. The shearing stress causes the sliding of the reinforcing bars in the concrete and increases the deformations, but in what degree cannot as yet be stated. New experiments will be required to determine the necessary corrections for sections subjected to shearing stresses in order to apply the formulas established on the assumption of conservation of planes and, hence, on that of the absence of shearing stress.

14. Effect of Surroundings in which Concrete is Placed.

Prism No. 34 was kept in water 210 days and was taken out only three days before the test, while structures built in the open air are tested after an almost complete drying out. To explain the important differences for the deformations which must result from the different surroundings, it is necessary to state briefly the effect of the hardening of concrete in air and water. It is well known that according as mortar or concrete has set in the open air or in water they shrink or swell. These changes of volume reach 0.2 per cent. for blocks made of pure Portland cement and decrease with the proportion of cement to sand and gravel, but without falling appreciably below 0.04 per cent. for the concrete poorest in cement generally employed on construction.

The metal reinforcing embedded in mortar or concrete tends to preserve its initial length, but the surrounding material adheres to it. A complex state of equilibrium is established in which the longitudinal sliding exerts a great influence, and compression results in the reinforcing bars and tension in the concrete, if it has been kept in the air and, on the contrary, tension takes place in the reinforcing bars and compression in the concrete, if it

has been kept under water. It is in order to determine the effect which these stresses have on the deformation of reinforced concrete beams. Let us consider two otherwise identical prisms one of which was kept in air and the other in water. Let us apply the same load to both prisms. As long as the elastic limit of the concrete be not exceeded, the deformations will be the same for the two prisms, because, if even the absolute values of the stresses produced in the metal and concrete be different, their variations will be identical, and in the elastic period the deformations depend only on the variations of the stresses and not on their absolute values.

When the elasticity of the concrete changes, the deformations of the two prisms will, on the contrary, be very different. To illustrate this more easily, we shall neglect the intermediate period and consider the period when the elongation of the concrete in tension exceeds at all points the elastic elongation, except in a practically negligable portion nearest the neutral axis. If, for simplicity's sake, the slight difference in the position of the neutral axis in the two prisms be temporarily neglected, with the intention of returning to it, the following reasoning will be true. Let M be the moment to which each of the two prisms is subjected, and μ the moment of resistance produced by the tensile stresses in the elongated portion of the concrete and an equal compressed portion, which completes the resisting couple. Then μ will be of the same value in both prisms, because the elastic elongation being, by assumption, exceeded, the portion in tension will cause in one as in the other prism the same tensile stress t. The reinforcing metal in the two prisms must, for equilibrium, produce the same moment $M - \mu$, and consequently it must take the same absolute elongation λ. But in the prism kept in the air the reinforcing bars were compressed by the shrinking of the concrete and shortened by the amount r before the load was applied, while in the prism kept in

water they had elongated by the amount d. Consequently to obtain the same absolute elongation λ, the reinforcing bars of the prism kept in air must elongate by the amount $\lambda + r$, while those in the prism kept in water must elongate by $\lambda - d$ only. Generally the values of r and d are from 0.01 to 0.02 per cent. of the length of the member.

Such is not exactly the case because of the displacement of the neutral axis which shifts farther away from the reinforcing bars the more these elongate. In prisms kept in air, an increase in the lever arm of the couple resisting bending is the result, and hence a decrease in the tensile stress required to produce the moment M and the corresponding elongation of the reinforcing bars. The opposite evidently takes place in prisms kept in water, so that really the variations of length which the test load causes in the reinforcing members are somewhat less than $\lambda + r$ for the former and somewhat more than $\lambda - d$ for the latter.

But without injuring the beams by holes in the concrete the variations in length of the reinforcing bars cannot be measured, and the concrete only is measured.

The tendency for longitudinal sliding, which the shearing force produces in the reinforcing bars, has a direction opposite to the one caused by the shrinking in beams kept in air, while in the prisms kept in water it has the same direction as that due to the swelling. This is a new cause of the relative increase in the deformations of concrete, and hence, of deflections of beams kept in air. It is easily understood from the preceding that transverse reinforcing bars which decrease the sliding tendency must hence reduce the deformations of structures of reinforced concrete.

It has been seen how many elements have an effect on the deformation of reinforced beams, and how numerous investigations must be made to determine exactly the rolé and importance of each. However, the conceptions ac-

quired seem to be sufficient to obtain practical results, with the condition of making on each structure some tests of normally built members, under the supervision of the engineer, with the materials and form of reinforcing used in the structure. From the results of these tests the modifications will be deduced which must be applied to the figures given by the formulas, based on the assumptions of the conservation of planes and the absence of initial internal stresses, in order that their results should agree with the actual deformations of the beams built of the same materials and with the same care as for the structures.

15. Informations Obtained from Tests.

The chances for poor workmanship in reinforced concrete construction are numerous, and poor sand or a low grade cement may reduce all the qualities of the concrete. The proportion of the water used exerts a considerable influence on the coefficient of elasticity, which may fall to 2,100,000 pounds for materials from which 3,500,000 pounds can be obtained by using a proper proportion of water. The resistance also decreases but in a less proportion than the coefficient of elasticity, if there be an excess of water in the concrete. The tamping is of no less importance, and insufficient tamping seems to reduce the resistance and the coefficient of elasticity in pretty much the same degree.

It may also happen that the proportion of cement prescribed by the specifications is not observed. Should this occur the tensile and compressive resistances will be decreased as well as the coefficient of elasticity, but from numerous experiments it is known that the coefficient of elasticity will be decreased in a smaller proportion. The study of the curves of deformation permits us to distinguish the different modifications which characterize

the properties of concrete. In fact, during the perfectly elastic period under light loads the deformations depend exclusively on the coefficient of elasticity. By comparing the inclinations to the horizontal of the computed and actually observed lines O H and O′ H′, in Figs. 9 and

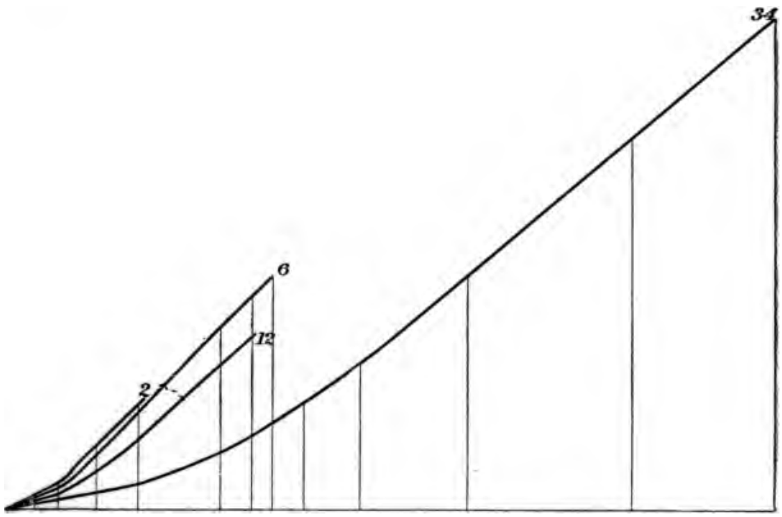

FIG. 11.

Prism No. 2, 500 pounds of cement, one cubic yard sand and gravel equal parts. Excess of water. Seven months in air.
Prism No. 6, 500 pounds of cement, one cubic yard sand and gravel equal parts. Insufficient water. Seven months in air.
Prism No. 12, 500 pounds of cement, one cubic yard sand and gravel equal parts. Normal amount of water. Seven months in air.
Prism No. 34, 720 pounds of cement, one cubic yard sand. Normal amount of water. Seven months in water.

10, which represent the elastic deformations, indications will be obtained as to the value of the coefficient of elasticity. As to the elastic limit, it is proportional to the abscissas of the points H and H′, where the inclination of the curves changes almost suddenly.

The abscissas of the lines O D and O′ D′ represent, as has been shown, the deformations which would have taken place if the concrete could resist no tension. The portions of the abscissas between these lines and the curves of observed deformations O B and O′ B′ are thus almost proportional to the actual tensions, which are nearly identical with the breaking resistances in tension. From the curves of deformation and the variation of the position of the neutral axis, information can also be deduced as to the sliding of the reinforcing metal in the concrete, but the study of this question would lead too far from the present subject. It would involve still more time and space to investigate the precise effect which the causes modifying the deformation of reinforced concrete beams have on the stress and fatigue of each of the constituent elements.

It will be sufficient for the present purposes to illustrate the differences caused in the deformation of reinforced concrete beams by reproducing the diagrams of the elongations of the concrete or mortar of the tension side of four prisms. These prisms were identical in dimensions and reinforcing with prism No. 34, which was one of the four, that is, they had a square section of 2.36 inches on a side and were reinforced by three wires 0.17 inch in diameter. Three prisms, Nos. 2, 6, and 12, contained 500 pounds of cement to the cubic yard of a mixture of sand and fine gravel, the proportions employed by M. Hennebique. The fourth prism, No. 34, contained 720 pounds of cement to the cubic yard of sand, which the author believed to be equivalent; but these proportions may possibly be somewhat too rich. The first three prisms differed only in the intentional excess or insufficiency of water. The deformations were the smallest in the prisms made under the most normal conditions.

The difference in deformation between the prism No. 34 and the others is very great, and is in a measure due to the difference in the coefficients of elasticity of the mortar

or concrete, but still more to the fact that the first three prisms were kept in the air and the last in water. These examples will demonstrate that the differences which practical load tests will show between reinforced concrete beams will be great enough not to require measuring instruments of great precision.

16. The Deformations of Beams Symmetrically Reinforced.

In the preceding sections of this chapter beams reinforced on the tension side only have been discussed, but beams reinforced also on the compression side are sometimes used. Their deformations can be easily determined by means of the diagrams which have been established for beams reinforced on one side only. Let A B in Fig. 12 be the cross-section of a beam reinforced by rods F on one side only. If this section be subjected to a unit bending moment m, the neutral axis will be in the position O, which can be determined by means of equations (7) to (10).

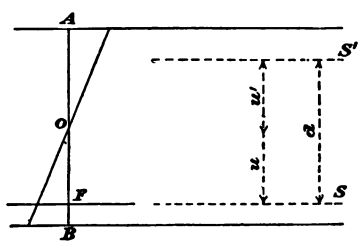

FIG. 12.

If to this beam a supplementary reinforcing area of steel S in tension, and one S′ in compression be added, and if the distances from the neutral axis be denoted by u and u′, the total stresses of opposite signs which will be produced in these new reinforcing rods will be equal to

each other, provided the areas of the additional reinforcing satisfy the conditions $S u = S' u'$, because the stresses which the reinforcing will sustain per square inch will be proportional to the deformations, and, therefore, to the distances from the neutral axis, u and u′. The equilibrium of the beam with reinforcing on one side only under the moment *m* will thus not be disturbed, if at the same time with the addition of the new reinforcing rods, the moment of the external forces also be increased by an amount equal to the moment of resistance furnished by the tension in S and the compression in S′. The total value of these forces is easily determined, because the additional reinforcing metal in tension, being placed at the same distance from the neutral axis as the reinforcing steel of the initial beam, will sustain the same elongation and, consequently, also the same stress per square inch *s* as the steel in this initial beam. This stress *s* is given by the formulas (7) to (10) or by the tables or diagrams established by means of these formulas.

The additional moment of resistance is hence:

$$S\ s\ (u + u') = S\ s\ d,$$

where *d* is the distance between the reinforcing members. Consequently nothing is changed in the unit stresses in the deformations caused in any part of the steel or concrete of a beam reinforced on one side only if to both sides reinforcing rods be added the sectional areas of which are to each other as $u : u'$, and if the bending moment $S\ s\ d$ be increased at the same time. The ratio $u : u'$ is deduced directly from the position of the neutral axis, that is, from the value of x, which is given by the formulas (7) to (10).

Thus, in order that the diagrams or tables prepared for beams reinforced on one side only should also furnish the necessary information for beams reinforced on both sides, with any reinforcing members, it suffices to add to them a concurring table.

6

Let γ represent the value which the ratio $u:u'$ assumes under the test load for a beam reinforced on one side only by the proportion of metal to concrete p. The deformations computed for this beam will be exactly the same as for all the beams reinforced on both sides by additional reinforcing members, the sectional areas of which will be in the ratio γ, provided that they be subjected to the bending test increased by $S\ s\ d$.

Let us, for an example, assume that for beams reinforced on one side by 2 per cent. of metal, the value of γ for the test load will be 1.10. The deformations of these beams will then be the same as those of all beams in which the reinforcing rods in tension have an area of $0.02 + y$, with a reinforcing in compression of $1.10\ y$, where y has any value whatever. Thus the same table will be applicable for all beams, the double reinforcing of which will have cross-sectional areas given by the following figures or interpolations between them:

Reinforcing in tension, per cent.,	2	2.5	3	3.5	4.
Reinforcing in compression, per cent.,	0	0.55	1.1	1.65	2.2

It is needless to explain here in more detail the very simple methods of interpolation by means of which, once these concurring tables have been established for the different percentages of single reinforcing, the corresponding tables for double reinforcing of any sectional area can be deduced. It should, however, be understood that the values of the unit moment of resistance will be replaced by $m + S\ s\ d$. It is hardly necessary to state that by means similar to those just indicated for the computation of the deformations of beams reinforced on both sides, the strength of these beams can also be computed.

17. The Computation of Deflections.

It has been shown that tests based on the measuring of the elongations and shortenings which take place in cer-

tain sections of beams in bending readily furnish useful information on the quality of the materials of which they are made, and it is probable that engineers who are provided with the necessary instruments for the observations will make use of this method.

The deflections of prisms loaded as beams are due to the deformations of an indefinite number of sections, and it is, from the beginning, evident that by observing this complicated phenomenon it will not be possible to obtain as easily such accurate results as by the study of the deformation of the sections. However, since the measuring of the deflections is of almost general use and does not require any special instruments, it is probable that it will continue to be used frequently, and it should here be mentioned, though somewhat hastily.

To compute the deflections δ of reinforced concrete beams it is convenient to make use of the rarely used formula

$$\delta = \int_0^1 \frac{l-x}{\rho}\, dx$$

where ρ represents the radius of curvature of the beam in flexure at the distance x from the center of span, and l one-half of the span. ρ being given directly as a function of the elongation λ of a "fibre." The shortening r of another, distant i from the first, is given by the relation

$$\rho = \frac{i}{\lambda + r}$$

The deflection thus becomes:

$$\delta = \int_0^1 \frac{\lambda + r}{i} (l - x)\, dx$$

The quantities i and l are known and the values of λ and r are given by equations (7) to (10) as functions of the moment m to which the different sections of the beam are subjected. The direct integration of this formula is impossible, but it will be easy to so find the approximate

value of δ by dividing the half span l into a convenient number of parts Δx, and computing for each of them the value $\frac{\lambda + r}{i}(l - x)$.

The computation will be easy, but it will have to be repeated for each type of beam, and there are no evident reasons for accepting these complications when a single set of tables can be established, including tables applicable for different qualities of concrete. These tables will give directly the elongations and shortenings caused in any cross-section of any beam made of given materials.

It should be noted that the forms of the curves plotted with the bending moments as abscissas and the deflections as ordinates are notably different from those of the curves of elongations or shortenings discussed in this chapter. The latter form almost straight lines outside of the elastic period of the concrete in tension, at least they do not begin to curve except under very heavy loads, sufficient to modify the elasticity of the concrete in compression, if this modification takes place before the beam fails from other cause.

It is evident that this cannot be otherwise, because the moment of resistance to bending produced after the change in the elastic behavior of the concrete in tension and before that of the concrete in compression is equal to the sum of the two elements, the law of variation of which is linear. These two elements are the constant moment produced by the concrete in tension and the moment proportional to the deformations produced by the elastic metal and the concrete in compression as long as it is sensibly elastic. In numerous experiments the author has always found this long line to be almost straight.

It cannot be the same for the deflection curve, because as the load increases the elastic limit is exceeded in new sections which accelerates the increase of deflections. But this increase in deflection becomes less and less rapid as the beam approaches rupture.

18. Limits of Test Loads.

It appears to be evident that the test loads should not be heavier than the amount required to insure the safety of the structure. Even smaller loads could, no doubt, be employed if a comparison of the computed deformations and those actually observed is made, as has been proposed in this chapter, so as to obtain by means of moderate test loads the coefficient of elasticity, the elastic limit and the tensile resistance of the concrete of the tested beams. It is, besides, known that the compressive resistance is almost proportional to the tensile resistance and it is consequently evident that a test with a moderate load will suffice to furnish information on all the properties of the concrete employed.

19. Conclusions.

To the preliminary tests of the materials to be employed the direct tests of reinforced concrete structures must be added to obtain sufficient assurance of safety.

The tests will not show their full usefulness unless the deformations which should normally be expected be first computed and then compared to the observed deformations.

The computation of the deformations to be expected must be based on the knowledge of the laws of deformation of concrete reinforced by metal, and it appears that above the elastic limit, these laws are different from the laws which determine the deformations of unreinforced concrete.

The deformations of members in flexure appear to be influenced by the shearing stresses, by the character of the surroundings in which the concrete has set and was kept and by the action of any transverse reinforcing members.

The investigation and study of these phenomena is still more important because the strength of a member is intimately connected with its deformations.

It is easier to compute the elongations and shortenings which should be expected to take place, under normal conditions, in a given section than the deflection of a beam, which is the resultant of the deformations of all of its sections. The measuring of the local deformations deserves, therefore, to be recommended and at least to take its place side by side with the measuring of deflections.

CHAPTER III.

Effects of Changes in Volume of Concrete.

1. Shrinking of Concrete in Air.

As mentioned in the preceding chapter, mortar and concrete kept in the air have a tendency to shrink. The shrinkage is the greater the richer the concretes are in cement, and it varies from 0.15 to 0.2 per cent. of the length for neat cement, and from 0.03 to 0.05 per cent. for mortar poor in cement. When new masonry is built on a good resisting foundation or on masonry which has already had time to harden, the shrinking of the new material is hindered and tensile stresses parallel to the

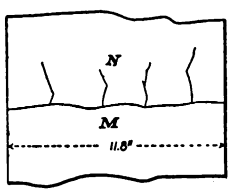

Fig. 13.

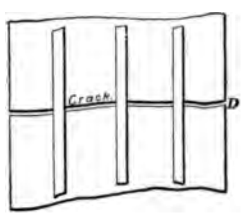

Fig. 14.

joints develop. In cubes 11.8 inches on a side, made from a batch of neat cement, these stresses caused numerous vertical cracks which did not appear before the lapse of

a year. Such cracks are shown in Fig. 13, where the portion denoted by M was laid three days before N. No cracks, however, were observed on identical cubes made of mortar containing less than 1,340 pounds of cement per cubic yard of sand.

In prisms made of neat cement with iron rods tying the blocks together, as shown in Fig. 14, and preventing shrinking in the vertical direction, the joints opened so much as to become visible to the naked eye. When the internal stresses, due to the shrinking of the concrete, did not cause cracks, they, nevertheless, changed the qualities of the mortar and concrete for the worse. An examination of the broken parts of a seapost of neat cement, 16.4 feet in diameter, which was smashed by the sea, showed in the portion above highest tides serious damage which could only be explained by the bending stresses produced by shrinking.

When shrinking is prevented not by external causes or by internal stresses in the concrete, but by reinforcing rods embedded in the mass, the internal stresses which are produced in the same are distributed somewhat uniformly by the action of the reinforcing rods; and, in fact, direct cracks have never been observed in any of the tests made, whatever the proportion of the concrete. But when prisms of neat cement, kept in dry air, were subjected to a bending test they cracked with elongations of 0.01 to 0.025 per cent., while similar prisms of mortar or concrete (500 to 1,000 pounds of cement to the cubic yard of sand and gravel) sustained, without cracking, elongations of 0.08 to 0.2 per cent. of length. This fact deserves attention, since neat cement, without reinforcing, elongates before breaking much more than common concrete. The inferiority of neat cement in reinforced members exposed to the air appears to be caused by its tendency to shrink four or five times more than concrete not rich in cement. The reinforcing rods, while preventing

the shrinking of the neat cement members, exhaust, to a large extent, the capacity of the cement for elongation, or counteract it completely.

When the reinforcing rods prevent shrinking and do not cause cracks, as is the case with common concrete, the stresses and changes due to the shrinking cause a decrease in elasticity, resistance and capacity for elongation before rupture, which is the greater the higher the resistance which the reinforcing members offer to shrinking. These unfavorable changes in concrete increase with time during several months.

The practical consequences of these facts seem to be the following: The use of concretes of rich proportions of cement involve some dangers for masonry work exposed to the action of dry air, whether it be reinforced or not. Any means of preventing or hindering the shrinking of the concrete reduces the quality of the mortar or concrete in a measure which is the greater the richer its proportions are and the more effectively its shrinking is prevented. The disadvantages which result therefrom are of relatively small importance for concrete not rich in cement, and they are of great importance for mortars rich in cement and kept in dry air. It is, therefore, of advantage to maintain concrete masonry in a moist condition as long as possible.

2. Swelling of Concrete in Water.

Concretes placed under water and gradually hardened there show a tendency to swell in all directions, and the more so the richer their proportions of cement. This swelling varies from 0.1 to 0.2 per cent. for pure cement mortar and from 0.02 to 0.05 per cent. for concrete poor in cement.

When the swelling cannot take place freely, compressive stresses are developed in the masonry, which may

reach much higher values than the stresses caused by the shrinking in the air. Experiments were made which were intended to establish the law of relation between the deformations, prevented from exceeding certain limits by external means or metal reinforcing, and the stresses developed at the same time. An idea of the importance of these stresses will be formed by the fact that, in a rectangular prism 2.36 by 0.98 inch, made of neat cement and reinforced in its axis by an iron rod 0.4 inch diameter, there have been developed, after ten months in water, internal and opposing stresses of about 2,200 pounds, equivalent to a compressive stress of about 90 pounds per square inch in the concrete and a tensile stress of about 17,500 pounds per square inch in the iron. The tension in the iron was measured directly, and computed as accurately as possible by multiplying the coefficient of elasticity by the shortening of the rod caused at the instant when the surrounding cement which prevented it from taking its natural length was carefully removed.

The internal stresses caused by the prevention of the swelling of mortar or concrete members by the reinforcing rods are, in general, favorable to their resistance, because these stresses increase the compressive stresses and decrease the tensile stresses of the materials which can resist the former ten times better than the latter. They have especially the effect of consolidating building joints and all sections of small resistance to tension by preventing cracks in them. Obvious advantages result therefrom for the resistance of masonry kept under water and the durability of the concrete and its reinforcing metal.

Differences in the swelling of layers of different age must, however, be guarded against as they cause parallel stresses in the joints, which seem to be injurious to the adhesion. But, contrary to what takes place in the case of shrinking, it is here the oldest masonry in which tensile stresses are caused, and its resistance being superior

to that of the superimposed masonry the possibly dangerous effect is decreased. However, the author has no experimental proof as to the dangers due to the different swelling of parts of masonry. In general an exaggerated account of the internal stresses should be avoided because their effects combine according to little-known laws with the stresses caused by the external forces, and it may be possible that in some places their actions should be added together.

It seems, therefore, to be proper not to raise the proportions of cement in concrete above the limits which insure sufficient impermeability and long life to the submerged concrete. It appears to be of advantage not to exceed 1,300 to 1,500 pounds of cement to the cubic yard of concrete, which proportion gives the greatest resistance, except for work exposed to the waves, where rapidity of setting is a necessary condition for successful work. From all the above considerations it follows that reinforced concrete masonry will give still better results for hydraulic work than for structures exposed to the air, and the success of these has been proved by experience.

It should be stated that, in the computations of the resistances of concrete masonry structures in which the free change of volume is in any way prevented, these changes should be considered. On this basis the author recently showed that in the floor of a dock only harmless stresses could be produced in spite of the fact that this floor was of a thickness which would have caused excessive stresses if the increase in volume had not strongly compressed the floor against the foundations of the side walls.

3. Effects of the Proportion of Moisture.

It is well known that all the materials employed in masonry show an increase in volume when they absorb water and a decrease when their moisture is reduced.

The author has found these changes in volume to be much greater than is given by Busing and Schumann in their work on cement. A prism of neat cement, not reinforced, which was kept in dry air during two years elongated 0.024 per cent. of its length after three weeks of immersion in water. A prism of mortar, not reinforced, containing 730 pounds of cement to the cubic yard of sand, which was kept fifteen months in water, shortened 0.05 per cent. after being two months in dry air. From the observations made it appears that contrary to what has taken place in the changes of volume due to the gradual hardening of the cement, the variations in volume produced by changes in the state of moisture on hardened mortar do not increase with the higher proportions of cement. The contrary rather takes place.

There is still another radical difference between the effects which the two causes of change in volume have on reinforced concrete members. During the hardening the mortar possesses at the beginning a very high degree of plasticity, which gradually decreases. This results in causing the mortar, which has gradually hardened, to yield to a large extent to the stresses which the reinforcing rods produce in it. Thus reinforced members which have set in the open air remain of a greater length than that which they would have taken freely without restraint from the reinforcing. The crystallizations which take place during the setting are between the artificially separated molecules, and a decrease in density, elasticity, and resistance is the result.

The opposite should be caused in reinforced members which have hardened in water, but the author has not had the occasion to verify whether an improvement in the quality of the concrete actually takes place. The effects of the variations in the hygrometric condition on completely hardened mortars are very different from those resulting from gradual hardening. There is a struggle be-

tween the two associated materials, the coefficients of elasticity of which have arrived at their final values, and the difference between the length which each material tries to assume and the one it is compelled to take by the combination is inversely proportional to its coefficient of elasticity.

From these considerations it follows that account should be taken of the fact that the difference in the variations of length which take place in a mortar according to whether it is reinforced or not will be considerably greater during the beginning of the hardening than during the following hygrometric changes. Experience has confirmed this fact. It also follows that the variations in volume which the hygrometric changes tend to produce in hardened mortars are smaller than those produced during gradual hardening, since internal stresses, and especially tensile stresses in the reinforcing rods, may be caused, which attain 5,500 to 8,500 pounds per square inch. Contrary to what takes place during the slow hardening of mortar, the hygrometric variations appear to be the more dangerous the less rich in cement the mortar is, because its resistance is smaller while the internal stresses are, at least, as great.

It should be added that these disadvantages are practically of no account for masonry always exposed to the air because the changes in the proportions of moisture in the air have a very small effect. The question is only of importance for members made in the open air which begin to harden before being put in water where they finally remain submerged, as in the case of piles, caissons, etc. By keeping them moistened until put in place, not only will the cracks which are often caused, as has been shown by experience, be prevented, but also the changes in the internal stresses, which cannot be of any advantage. It would be both interesting and useful to determine experimentally the results to be obtained by keeping reinforced

concrete members, to be permanently exposed to the air, as moist as possible by abundant and repeated sprinkling for several weeks. It is obvious that the final shrinking, as well as the disadvantages resulting therefrom, would be decreased.

In concluding attention should be called to a fact worthy of research. Reinforced concrete members previously subjected to test loads have shown very little effect due to changes of the moisture in them. This will seem probable when it is remembered that the test load reduces the coefficient of elasticity of the concrete very considerably.

CHAPTER IV.

Tensile and Compressive Resistance of Reinforced Concrete.

1. The Tensile Resistance of Reinforced Concrete.

In Chapter I the bending experiments made by the author on reinforced concrete prisms were fully discussed. Some objections were raised against them which had the semblance of being justified. In these experiments the deformations of the mortar were measured, and in order to determine from them those of the reinforcing metal the generally accepted assumption of the conservation of plane sections had to be made. The greatest care, however, was taken to justify this assumption by limiting the observations to the central portion of the prisms where the shearing stresses, which are the ones to distort the plane sections, vanish. To avoid any uncertainty on this point — the only one open to dispute — new tension and bending experiments were subsequently made, in which the elongations of the reinforcing rods were measured directly and simultaneously with the deformations of the mortar.

The tension experiments especially are simple and seem to be free of error. Prisms made of mortar 1.85 by 1.85 inches in cross-section, were symmetrically reinforced by four wires of 0.17 inch diameter and subjected to a direct pull. For each load the elongation of the reinforcing wires λ and that of the mortar λ_1, were measured. As shown in Chapter II, the stress per square inch in both the wires and the mortar can be easily deduced from these observations. In a similar manner the stresses in the bending experiments were determined. Except measuring the compressed fibres of the mortar,

the computations required for the determination of the position of the neutral axis were of a more complicated character and admitted only very small inaccuracies, which were, however, too small to affect in any way the conclusions resulting from the experiments. The laws established on the basis of the first series of experiments, Chapter I, have been fully confirmed, supplemented, and also extended to the effects of repeated loads.

To illustrate easily and precisely the phenomena observed in the numerous and different tests made, the diagram shown in Fig. 15 was plotted for the simplest case, that of direct tension. The total loads applied suc-

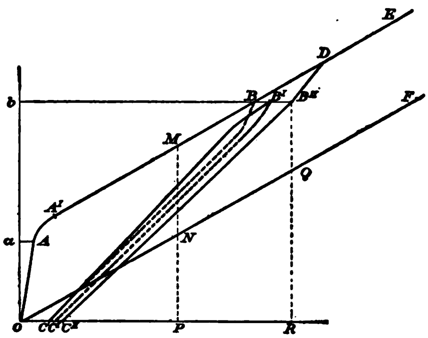

FIG. 15.

cessively on the prism were plotted as ordinates and the corresponding elongations of the reinforcing rods as abscissas. The elongations of the mortar, also measured directly, were found to be sensibly equal to those of the iron in the middle portion of the prisms subjected to simple tension, as well as in the portions of prisms in flexure which are sufficiently distant from the fixed ends and, therefore, free of shearing stresses. As long as the load did not exceed a certain value O a, the elongations increased regularly and were very small, then the increase

became almost suddenly much greater, but soon assumed a regular course, represented by the straight line A B. The load was limited to O b, then gradually reduced to zero. The resulting shortenings are represented by the dotted line B C which is almost a straight line except at the ends.

A series of new applications of load to O b, and corresponding unloadings caused the deformations represented by C B′, B′ C′, C′ B″, B″ C″, etc., that is by curves more and more distant from the vertical axis O b and less inclined to the horizontal. The alternate loading and unloading was continued until the final state was established, and it was observed that the points B and C approached more and more slowly the limits C^n B^n. It was also observed that in prisms where there were no preliminary stresses caused by the gradual hardening in air or water, the final value O C^n of the permanent elongation was always less than one-fifth and often less than one-tenth of the total elongation under the load O b.

To illustrate more precisely the other characteristics of the results obtained, it is necessary to plot the curve which represents the tensions taken by the reinforcing rods under different loads. This curve is evidently the straight line O F, the inclination of which to the horizontal is the product of the cross-section of the reinforcing rods by their coefficient of elasticity. If M be any point on the curve of deformation, N P represents the tension taken by the reinforcing rods when the reinforced prism sustained the total load M P, and consequently the tension resisted by the mortar was M N.

The observations to which the study of the results leads are the following: During the first period O A, where the elongation a A has not much exceeded the elongation which a prism of the same mortar, but not reinforced, will sustain without breaking, the tensile resistance of the reinforced mortar is the same as if not reinforced. The

two combined materials act as if independent and divide the stress proportionately to their coefficients of elasticity. Above this limit the elongations of the prism become much more rapid, the tension M N of the mortar increases only very slowly and hence its coefficient of elasticity becomes very small.

It is especially important for practical purposes to determine clearly, for the case of repeated loads, the final value of the resistance B^n Q which the mortar will sustain when subjected to an infinite number of repetitions of load without exceeding the limits O b and Q R. This is the difference of the stress existing at the same time between the mortar and the reinforcing metal. In prisms which were kept in the air and which were subjected during the test to a maximum elongation b B^n of 0.09 per cent. of the length the final value of the tension B^n Q resisted by the mortar was equal to 70 per cent. of the greatest tension M N and remained somewhat higher than the tensile resistance of the same concrete not reinforced. In prisms which were kept under water and then in the air for several days before the test so as to eliminate the internal stresses due to gradual hardening, the final tensile stress was higher than 70 per cent. of the greatest tensile stress.

Considering the elastic behavior of the prisms, the phenomena which take place during the tension of reinforced concrete lead to the conclusions which have already been formulated in Chapter I, for the tension portions of reinforced concrete in flexure. They hold true for both cases. After these conclusions it is certainly of importance to determine what would take place if, due to an accident, a reinforced concrete member be subjected to a load greater than that of the repeated test. The law of the phenomenon which would take place is represented by the curve B^n D E. It shows that in this case the concrete regains, after a certain increase in load, the greatest resistance which it had before the repeated applications of

load. This is an important property which would prove valuable in some cases, as, for instance, in that of an accident. These new and original tests throw some light on the behavior of reinforced concrete when subjected to a single application of a high load or to a repetition of loads. It seems that these properties could not be established either by theoretical reasoning or by the study of the results of previous experiments.

2. Effect of Shrinking and Swelling on Deformations and Stresses.

The laws established in Chapter I and in the preceding section are for reinforced concrete members which have been exposed to the air and water in such a manner as to reduce the internal stresses in them sufficiently to be neglected in the tests. In which direction the shrinking and swelling of the members will effect the stresses can be readily seen. It has been shown that as soon as the concrete exceeds a certain elongation, which is easily exceeded in all highly stressed reinforced structures, it offers almost a constant resistance to an increase in elongation. It follows, therefrom, that if an external force is applied to a reinforced concrete member, whether it be exposed to the air or water, the reinforcing bars will in any case offer the same resistance to deformation as the concrete and will thus have the same *absolute* elongation. The *relative* elongation will, of course, be greater in structures exposed to the air, as they were in initial compression before the test, and hence were shortened below their true length, while in structures under water elongations already exist. Hence the deformations must, under equal conditions, be greater in structures exposed to the air than in structures kept under water before being tested.

The effect of shrinking and swelling on the deformation of reinforced concrete structures is the greater the

greater the forces acting in them. In fact, the shrinking in the air causes the portions of beams in tension to elongate more than the portions in compression, and thus shifts the neutral axis toward the compression side, reducing its cross-sectional area. An increase in the unit stresses necessarily follows, since the concrete in compression tends to maintain equilibrium with the stresses in the reinforcing bars and in the concrete in tension.

Besides the shrinking of the concrete causes tensile stresses in the concrete surrounding the reinforcing metal and compressive stresses in the opposite portions. The shrinking thus has a double action to cause compressive stresses in portions opposite the reinforcing metal before the test and then to increase the compressive stresses caused by the test load. The effect of swelling is, of course, exactly the opposite. Experiments prove these conclusions.

3. Compressive Resistance of Concrete.

Generally the compressive resistance of the concrete is not increased by the imbedding of iron rods in the portions of reinforced concrete structures subjected to compressive stresses. It thus follows well-known laws to which the author's experiments have added nothing. However, the curve of deformation has been plotted for a prism in compression made of a mortar containing 1,000 pounds of cement to the cubic yard of sand, which was tested by the director of the laboratory of l'Ecole des Ponts et Chaussées after hardening 30 days in air. The curve, which is given in Fig. 16, shows the following important practical characteristics:

The shortenings are at the beginning proportional to the pressures and the coefficient of elasticity, hence, remains almost constant up to a load of about two-thirds of the ultimate resistance. The deformations increase with increased loads and attain relatively very high values.

It will further be seen that this property has an important effect on the ultimate resistance of concrete masonry in general and on reinforced concrete especially.

It is well known that mortar in thin joints can resist, without crushing, much higher pressures than can be resisted by prisms of considerable length. M. Harel de la

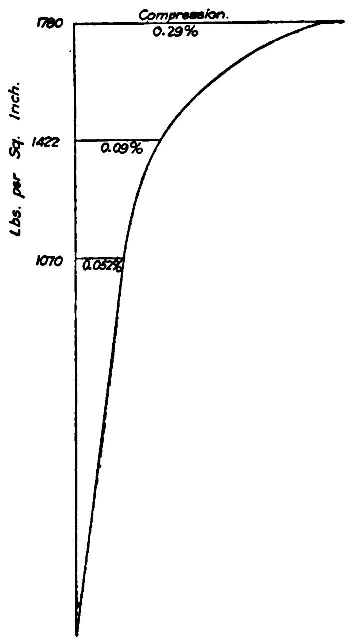

FIG. 16.

Noé has correctly concluded from these facts and theoretical investigations that the compressive resistance of concrete will be increased by reinforcing it with bars or rods in a direction perpendicular to the direction of the compressive force. Experiments confirm this deduction, but they also bring out a fact which should not be overlooked. (See Chapter VII.)

It has been seen that mortar and concrete have a tendency to reduce their volume when kept in the air. The reinforcing members prevent this contraction or shrinking the more effectively, the stronger and the more numerous they are and the more directions in which they spread. Hence it is probable that the stresses thus produced during the gradual hardening of the concrete will cause stresses in it which will act in a sense opposite to the compressive stresses and have a favorable effect, as has been pointed out by M. Candlot.

Comparative tests on two prisms, one of which was reinforced as usual by longitudinal rods in the tension side and the other also by additional cross rods in the compressed portion, gave the following results. The coefficient of elasticity of the portions in compression of the first prism was found to be twice as high as in the second. The volume of the reinforcing metal in the compressed portion was 1 per cent. of that of the mortar. These prisms, which suffered by the sliding of the longitudinal reinforcing rods, did not furnish any information as to the crushing resistance, but it is probable that they vary proportionally to the coefficient of elasticity, as generally occurs in mortar of the same proportion which is subjected to shocks or various compressive forces.

It would be of interest to carry out more numerous, systematic experiments on the subject, and to determine how much the reduction in quality, which the transverse reinforcing rods produce in the compressed concrete, is counteracted by the co-operating action of the resistance of the reinforcing rods themselves. This co-operating action is only of importance when the elastic limit has been exceeded. It follows that a similar but less pronounced reduction in quality also takes place in the concrete in tension which is reinforced by longitudinal rods. In all of the author's experiments, more than fifty, this was proved. The neutral axis was, in spite of the metal re-

inforcing, always found to be at the middle of the rectangular cross-section of reinforced members kept in the open air. It thus appeared as if the concrete had the same coefficient of elasticity throughout its whole mass, while the neutral axis, which generally shifts away from the less resistant parts, really ought to have moved nearer to the reinforcing, the coefficient of elasticity of which is, on the average, ten times as great as that of the concrete.

CHAPTER V.

Resistance of Concrete to Shearing and Sliding.

Though shearing and sliding show much similarity there are, however, essential differences between them. In shearing rupture is caused by sliding of the sections in directions which make an angle of 45° with the direction of the force, as has been clearly demonstrated by M. Mesnager. The reinforcing bars, on the contrary, which adhere strongly to the concrete, can be displaced only by sliding longitudinally, parallel to the longitudinal forces.

1. Shearing Stresses.

The experiments made by M. Mesnager tend to show that the resistance of mortar to shearing exceeds its tensile resistance as it is determined by the usual tests. Tests, which may be too few to allow of general conclusions, have shown a difference of 20 to 30 per cent. between these two resistances.

A fact recently observed has thrown some light on the ductibility of cement subjected to enormous shearing stresses at the same time with other complicated stresses, the compressive stresses much exceeding the tensile stresses. On a rock was placed a buoy made of a hollow iron pipe 7½ inches in diameter and filled with neat cement. The waves bent this buoy to a radius of curvature of about 22 inches, measured on the axis of the buoy, and it was expected to find the cement all pulverized. Cutting the buoy along its bent axis it was found that the cement showed some sliding surfaces only between which solid pieces were found, the deformation of which indicated a sliding of the "fibres" on each other of at least 20 per cent. of their length.

It must be concluded that, the same as reinforced concrete in tension, mortar or concrete can sustain enormous sliding when it is compressed, as it was in the interior of the above-described buoy. This fact, which is of importance for the resistance of masonry in deep foundations subjected to high pressures, does not seem to be so for the study of reinforced concrete construction. However, compared to the observations made on the sliding of the reinforcing members, to be described later, it will make them more complete and lead to conclusions which are not without importance.

2. Sliding Deformation and Resistance.

The author made numerous experiments on the sliding of the reinforcing metal by different methods, but all having as the object the measurement of the longitudinal displacements of certain points of the bars, relative to the adjacent surfaces of the prism which were in the same transverse section before the application of the load. The

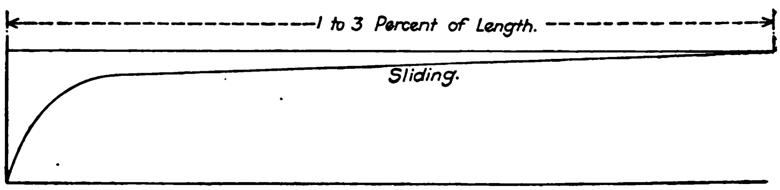

Fig. 17.

results obtained from different prisms were very different in their absolute values, but when they were represented graphically by diagrams, with the loads as ordinates and the corresponding sliding as abscissas, the uniform aspect of the deformations was seen clearly, Fig. 17.

The relative sliding or displacement of points of the reinforcing rods and the surrounding concrete, which were

distant from each other by the small distance of 0.2 to 0.3 inch, were extremely small as long as the sliding stresses did not exceed certain values, which evidently correspond to the elastic limit. Then, almost suddenly, the sliding increased rapidly, and for an increase in load of about one-tenth to one-fifth, at the most, they attained relatively considerable values, corresponding to a sliding from 1 to 3 per cent. The similarity of the form of the curves, represented by Fig. 17, to that of the curve of deformation of concrete in tension, which differs so much from the results on concrete not reinforced, is evident. But the values of the deformations are much higher for sliding than for tension, possibly ten or fifteen times as much. This result would seem to be doubtful if the examination of the above-mentioned buoy had not revealed still greater sliding.

It should be observed that these sliding curves, which have a very definite practical value, do not have a precise scientific explanation. In fact, for reasons which need not be given here, the stresses per square inch of surface of contact which tend to produce sliding of the reinforcing rods are not at all proportional to the applied loads taken for the ordinates of the curve in Fig. 17. On the other hand, these unit stresses vary with the distance from the reinforcing rods, since cylindrical rings of mortar are considered on the circumferences of which the stresses are distributed. The observed sliding is thus the complex resultant of different displacements which occur in the variously stressed portions. This lack of scientific precision does not diminish the importance of the observations on the considerable ductility which mortar or concrete possess as to sliding, and which increases with the increase in pressure on the concrete.

The observations of the author cover not only the deformations, but also the resistance of the reinforcing rods to sliding. For reasons similar to those for the deforma-

tions, their useful practical results cannot establish the elementary laws of the sliding phenomenon. It is, however, of no less interest for practical purposes to know the results. Prisms were loaded the same as the prism described in Chapter I, until rupture by the sliding of the reinforcing members. In the absence of a better method the usual but, as seen, inaccurate assumption was made that sliding stresses are proportional to the shearing stresses, and it was found that sliding resistance varies from 70 to 170 pounds per square inch of surface of contact for prisms kept in the air and made of concrete containing 500 pounds of cement per cubic yard of a mixture of equal parts of good sand and small gravel. The reinforcing consisted of drawn iron wires of 0.17 inch diameter, the surface of which was perfectly clean, shining, and possibly somewhat greasy.

The resistance to sliding increased to 256 pounds for prisms of the same concrete, reinforced by rolled iron wires 0.24 inch in diameter, the surface of which was similar to that of the rods usually employed in practice. In another series of prisms made of mortar containing 730 pounds of cement to the cubic yard of sand and kept in water, reinforced by iron wires 0.17 inch in diameter and slightly rusted, the sliding resistance, computed the same as above, varied from 330 to 500 pounds per square inch of surface of contact.

It was determined, for mortars kept in the air, that the proportion of water used in mixing exerts a considerable influence on the adhesion to the reinforcing metal. In three prisms, otherwise identical, concrete mixed with an excess of water, normal concrete and too dry concrete were used, and the resistances to sliding were respectively 155, 170, and 70 pounds per square inch. These results agree with the opinion of practical men, and the weak adhesion to the reinforcing rods of concrete which is too dry is easily understood to be due to the

smaller number of points in contact. On the contrary, in concrete made by experienced workmen an excess of water gives to the cement the necessary fluidity to circulate between the grains of the sand and to fill all voids around the reinforcing metal. These advantages of wet concrete for adhesion are, however, counteracted by an appreciable decrease in the tensile and compressive resistances of the concrete, as has been shown by M. Candlot.

The results stated above should be verified by tests on masonry of usual dimensions, because concrete fails under altogether special conditions when the dimensions of the prisms are as small as the test prisms, which did not exceed 2.4 inches on a side. It will be noticed at once how much the sliding resistance as determined is below the values of 570 to 710 pounds per square inch, generally accepted according to the experiments of Bauschinger and Jolly. Part of the difference at least must be attributed to the fact that the reinforcing rods of a prism in bending are surrounded by concrete, which sustains at the same time very high tensile stresses, generally much above its elastic limit. The experiments of Bauschinger and Jolly, on the contrary, were made on metal bars well embedded in concrete blocks where all other stresses but the sliding could be neglected.

The results of the author's experiments on sliding seem to agree with the experimental results obtained by Hartmann, and the theory developed by Mesnager and others. Sliding, which has a deciding role in the deformation of the metals, and even in that of all other bodies, is resisted by two kinds of stresses: One is due to the strength of the material, the other to the frictional resistances, which are proportional to the normal pressures on the sliding surfaces. It is thus natural that the sliding and shearing resistances are influenced by all kinds of stresses which are acting in the reinforced concrete, and that they are higher in the parts closely adhering to the metal than in

reinforced concrete beams, the concrete of which has spent part of its resistance on the elongations due to bending, which have exceeded its elastic limit.

If it be also noticed that the sliding of the reinforcing members and the deformations of the plane sections increase the deflection, the conclusion is reached, which is contrary to the elementary and accepted notions on the resistance of materials, that the bending moments can have much effect on the resistance of concrete to shearing and sliding, and that the effects of the shearing stresses on the deformations cannot be absolutely neglected.

CHAPTER VI.

Effect of Cracks on Stresses and Deformations.

1. Effect of Cracks.

It has been assumed above that the reinforced concrete was in good condition and that the elongations due to the applied loads did not cause rupture. For reasons pointed out in Chapter III, this is frequently otherwise, especially in structures exposed to the air, and it is of importance to investigate the effects of the cracks which interrupt the continuity. Generally practical builders to avoid any errors as to the tensile resistance of the concrete assume this resistance to be zero. They determine in an arbitrary way the position of the neutral axis and, hence, also the resistance of the portion in compression, which are made a function of the assumed greatest stress.

It is easy to determine exactly the position which will be taken by the neutral axis since the concrete in tension has no effect. In Chapter I the formulas were reprinted, which led to this assumption. The following considerations will demonstrate that they do not hold true: In the first critical period, which is the period we have to look out for, only a small number of widely separated cracks are generally formed. The curves the ordinates of which represent the tensile stresses of the concrete and reinforcing members adjacent to one of these cracks show clearly the forms given in Fig. 18.

The tensile stress in the concrete which has the value a A in the uninjured sections decreases at the cracked section to zero. The tensile stress of the reinforcing steel, on the contrary, rises from b B to c C, since it tends to compensate for the gradually decreasing resistance of the concrete. The difference in the stresses in adjacent sections necessarily causes longitudinal action

due to the adhesion of the steel to the concrete, which thus causes sliding stresses, represented by the ordinates of the curve D E F in Fig. 18. Each force causes a relative displacement, the crack thus opens, the bending of the member at the cracked section shifts to the ad-

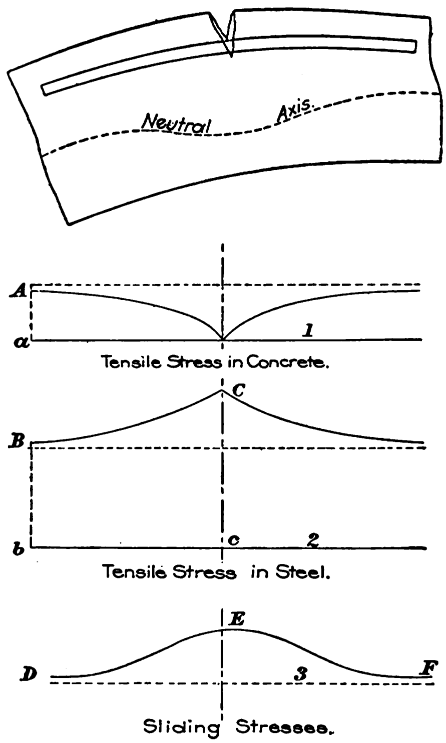

Fig. 18.

jacent sections and the neutral axis approaches the compressed side, the sectional area of which is thus decreased, while the stress per square inch is increased. This phenomenon differs according to whether the concrete is under water or in the air and, consequently, whether the

concrete adjacent to the reinforcing steel has an initial tension or compression.

It is doubtless true that the effect of the cracks will be the more prominent the greater the cross-sectional area of the reinforcing rods is to their surface of contact. Actually the differences in stress C c — B b which have to counteract the sliding stresses are proportional to the areas of the reinforcing members, while the adhesion which equalizes them is proportional to their area of contact. The greatest value of the sliding stress which causes a crack and the length of the displaced reinforcing are to each other as the cross-sectional area to the circumference of the reinforcing members. To verify these assumptions the following experiments were made:

Of a mortar containing 730 pounds of cement per cubic yard of sand twelve prismatic test pieces were made of 35.4 inches length and of a square cross-section of 2.36 inches. Three of these had no reinforcing and were intended to characterize the properties of the mortar alone. Each of the remaining prisms was reinforced with metal of a total area of 0.07 square inch arranged so that three prisms were reinforced by a single rod, three others by three rods, and the remaining three by five rods, which all had the same cross-sectional area.

A series consisting of one of each of these four types was kept all the time in the air and was tested after two months. A second series was successively and alternately kept in the air and under water so as to reduce the shrinking and swelling to the least possible and thus to avoid all initial stress before the tests. This second series was also tested after two months' time. The third series was kept under water, and it was the intention to keep it there until the swelling had such an effect as to show elongations in the reinforcing rods. In each of the reinforced prisms a slit was provided in the tension side by inserting a small and thin metal piece smeared with wax which

did not adhere to the concrete. The prisms of the first two series, as well as those of the third series, were subjected to bending moments which increased to much higher limits than would be allowable for practical load tests. The elongations of the mortar in tension were measured as well as those of the reinforcing rods and the shortenings of the concrete in compression. These measurements were made on lengths of 2.36 inches, equal to the depth of the prisms, on parts having the above-mentioned slits in their middle and also on parts of completely continuous and uninjured mortar.

The results were completely in accordance with the assumptions in the prisms of the second series where the internal stresses due to gradual hardening were almost of zero value. An increase in the bending of the reinforcing rods was found in them near the cracks, also a sliding of the iron on its surrounding concrete, and finally an increase in the shortenings of the mortar in compression. The intensity of all of these phenomena increased with the diameter of the reinforcing rods. In the prisms kept in the air the reinforcing rods were shortened during the gradual hardening process by 0.021 to 0.03 per cent. The effects of the slits in them were the same as for the second series, but with more intensity and have by far exceeded the author's assumptions.

In the prism reinforced by a single iron rod of 0.3 inch diameter the sliding of the rod relative to the surface of the mortar reached the value of 0.015 per cent., which is very near the ultimate stress. It should, however, be added that the diameter of the reinforcing rod of this prism was intentionally made much larger than it would be made in practice, to illustrate more clearly the effect of the dimensions of the reinforcing metal. But the most surprising fact was the amount of the shortenings of the compressed portions. In the prism reinforced by three iron wires of 0.17 inch diameter, where

the dimensions of concrete and iron agreed with tried practical cases, the shortenings near the slit were found to be ten times as great as in the uninjured portion of the prism, though the load was a light one. It was 5 per cent. below a similar load admissible for load tests.

If the forces were always proportional to the shortenings it would have to be concluded that the crack or slit increased the compressive stresses in the portion opposite the reinforcing members five to ten times, which would give absurd values. The careful separation of the results of these experiments furnishes the explanation of this evident irregularity. These results prove that the coefficient of elasticity of the mortar decreased considerably at the cracked sections. Though the forces are proportional to the elastic deformations the compressive stresses at the cracks could not increase to a very high degree, in spite of the very great shortenings.

However it be, the change in the elastic behavior at the cracked section is above doubt, and the following remarks will throw some light on the cause of this change: The shortenings increased very much with the increase in diameter of the reinforcing rods though the cross-sectional area of the iron was the same in all prisms. But it seems that for equal areas the diameter can have an effect on the contraction only by the rigidity of the iron wires, and the rigidity increases rapidly with the diameter. It must, therefore, be concluded that the rigidity of the reinforcing rods, which offers a resistance to the contraction very different from that caused by the longitudinal stresses, had most of its effect on the sections weakened by cracks. And, contrary to what might have been supposed, the portions opposite the reinforcing rods maintained their stress during the contraction, and also underwent a change in their elasticity when the contraction was prevented, as was actually observed in the experiments. This is especially true of prisms reinforced by transverse reinforcing bars.

It should also be remarked that the above test prisms sustained no load during their hardening. It is easily possible that prisms sustaining permanent loads, as beams and floors, for instance, do not, after the removal of the falsework, undergo the same deformations as the test prisms, because the loads cause sufficiently high compressive stresses to equilibrate the rigidity of the reinforcing members.

In view of so complicated phenomena it would be presumptuous to predict the effect of cracks on the failure of a beam by basing these predictions on experiments on the elasticity of prisms which were not so conducted as to insure their crushing before the sliding of the reinforcing rods. Crushing experiments are also required to make the subject clearer. It seems that it is always possible to make with all due care certain assumptions which are based on the following considerations, the general value of which for the explanation of the facts observed on reinforced concrete structures has been established.

2. Effects of the Convexity of the Curves of Deformation of Mortar and Concrete.

It has been seen that the curves of deformation are convex toward the horizontal axis and that this convexity is quite marked for concrete in compression, but it is much stronger for the concrete in tension of reinforced concrete structures and for the sliding resistances of the reinforcing members. It follows that when a portion of the concrete has reached the elastic limit for any one of the deformations, the stresses which are caused thereby increase from then on only very slowly. The adjacent portions, on the contrary, which have been less deformed, sustain stresses which grow rapidly so as to compensate for the overstressing and to delay rupture.

This is the same with all elastic bodies. If a masonry

structure contains in addition bending stresses, the portions in tension and compression do not possess the corresponding resistances. The weakest portion undergoes a greater deformation than the rest. The neutral axis, therefore, shifts away from this portion and there results an increase in the cross-sectional area of the portions which must resist the greatest stresses. If the elasticity in a portion of the structure has been initially reduced because of contractions or any other cause, the deformations increase, the neutral axis shifts still farther away, and new portions come to the assistance of those having too little resistance.

It is thus always correct to consider the favorable effects of the mutual aid of the parts of concrete-steel structures as a very important fact. It is known that rupture may be caused by the repetition of smaller stresses, perhaps by two-thirds or one-half of the resistance of a single application of load which causes rupture. Especially is this true for loads the single application of which is up to the ultimate resistance. It is, therefore, so much more permissible to make assumptions, and it seems thus to be probable that cracks reduce the resistance of concrete-steel structures much more with repeated loads than with a single application of the greatest load. Experiments will settle this question.

Finally it should be stated that the above experiments refer only to the slits or cracks provided during the making of the test prisms. The cracks which are formed during the loading cause no alteration in the concrete. It is thus probable that the effects which result from them are also, for structures built in the air, similar to the effects which have been observed in prisms kept partly in the air and partly in water. The latter was done in order to reduce as near to zero as possible the internal stresses in the prisms and the reductions in elasticity corresponding to them. These effects will consequently be modified and much smaller than those observed on prisms

kept in the air only. The diagrams of Fig. 18 represent the results of these tests. They were presented before the members of the Testing Commission who found them to demonstrate the correctness of most of the laws formulated in this and in the preceding chapters.

3. Computation of the Dimensions and Deformations of Reinforced Concrete Beams.

From the facts observed in all of the above experiments a great complication of the different conditions follows. But these conditions must be known to understand the phenomena observed on structures erected and during test loads; hence, also the effects of the properties and proportions of the elements of which the concrete-steel structures consist. For the sake of simplicity in computation, the complications must be reduced; and to establish rules for computations, it is quite logical not to proceed with absolute rigor, since the properties of concrete are as variable as its materials and depend to a great extent on its careful mixing. It is only important to know exactly the errors made voluntarily in order to simplify the subject; and to determine the limits of these errors the laws governing the behavior of concrete-steel structures must be known.

In Chapters I and II it was shown that the knowledge of the curves of deformation of the concrete in tension and compression is sufficient to compute the stresses and deformations caused by the bending moments, if the following conditions are fulfilled: The concrete in tension must be unbroken and continuous; there must be no initial internal stresses before the loading; there must be no noticeable disturbances by the shearing stresses.

The rapid algebraic method of computation shown in Chapters I and II as a substitute for the exact method, which suffers from graphical inaccuracies, lacks elegance; and it will certainly be improved by other engineers,

but its results appear to be safe. To take the internal stresses into account, especially those caused by slow hardening in the air and under water, it will suffice to correct the results by certain coefficients. These coefficients will give somewhat higher values for the computed deformations and unit stresses of the concrete in compression and somewhat lower values for the stresses in the reinforcing metal of structures kept in the air. The opposite will be the result for structures under water.

Finally it would be very proper to assume cracks for all structures kept in the air and it would then be possible to judge when the same assumption should also be made for structures under water. The discontinuity of the concrete will then be taken into account by means of coefficients, which will actually increase the computed values of the tensile stresses in the reinforcing metal and the compressive in the concrete. As to the compressive stress of the concrete, an increase in the same would be the more important the larger the cross-sectional areas of the reinforcing members are to their circumferences. Thus the shifting of the neutral axis will be injurious to the increase in the tensile stresses of the reinforcing, but in so slight a degree that it is not worth the while to take account of it.

One of the results of these researches is that the engineer who knows the heterogeneous nature of concrete-steel structures will be able to judge and estimate the dangerous conditions. The experiments on the sliding of the reinforcing members and on the effects of cracks raise it above doubt that concrete-steel construction the more insures the strength and durability of the structure the greater the number and the smaller the cross-sectional areas of the reinforcing members relatively to their circumference. But it is easily seen that the great multiplication of reinforcing units has practical objections. It is very difficult to establish in each case the correct effect of the opposing conditions.

CHAPTER VII.

The Compressive Resistance of Reinforced Concrete and Hooped Concrete.

1. Concrete Reinforced by Longitudinal Rods.

The first idea which presented itself to engineers to increase the resistance of concrete in compression was to reinforce it, similarly to tension pieces, by rods laid longitudinally in the direction of the stress. For purposes of construction, to keep the rods better in place, the reinforcing rods were tied together by a network or a belt of smaller rods. Some engineers understood that these belts perform another important role, that they protect the longitudinal rods from premature flexure and retard the swelling of the concrete and, hence, its ultimate failure.

It will be seen below that by hooping or completely surrounding the concrete by steel rods a considerably higher resistance can be obtained, and it is evident that between this method, supplemented by the addition of longitudinal rods, on one side, and the method of reinforcing by longitudinal main rods tied together by belts of lighter material on the other side, there is an intermediate continuous series of methods of reinforcing. The conclusions reached by this study will enable us to foresee the effects of these complex combinations, but before making the synthesis the influence of each element should be investigated separately. Concrete reinforced by longitudinal rods tied together by netting or belts of dimensions too small or spaced too far apart to exert noticeable influence on the resistance of concrete will, therefore, be treated first.

It was admitted, up to the present time, that the different varieties of stone, mortars, and concrete, when under compression, always fail by shearing along planes

which are inclined to the direction of the stress. The recent experiments made in Germany by Foeppel and repeated by Mesnager at the laboratory of l'Ecole des Ponts et Chaussées have proved that this mode of failure is due to the friction exerted on the lower planes of the test specimens by the plates transmitting the pressure. And it has further been proved that by sufficiently reducing this friction by the introduction of a greased surface, the failure will take place along surfaces which will be parallel to the direction of the pressure.

It is not clear how longitudinal reinforcing bars, which are parallel to the lines of rupture, could prevent the separation of the molecules and increase the resistance of the concrete, and it seems that the only effect of longitudinal reinforcing in compression members consists in adding the resistance of the steel to that of the concrete without strengthening the latter. Experience has shown that such is the case. The effects of the reinforcing bars are, however, complicated, for the reasons which follow.

As has been shown in Chapter III, the tendency to shrink which concrete shows when hardening in air causes in reinforced concrete internal stresses of great intensity; tension in the concrete and compression in the metal. Experiments made in 1902 at the laboratory of l'Ecole des Ponts et Chaussées, according to the program laid out by the French Commission on Concrete-Steel, have determined the effect due to the shrinking of large concrete-steel specimens of the most commonly employed mixture, 420 pounds of Portland cement to the cubic yard of sand and 1-inch gravel in the proportion of 1:2. Measurement of the variations in length of the reinforcing bars has shown that after three months the shrinking of the concrete had compressed the metal, 6,540 pounds per square inch, in prisms 6.5 feet long of a section about 4x4 inches and reinforced near the edges by 4 iron wires $\frac{1}{4}$ inch in diameter. The compressive stress in the metal has reached 10,800 to 14,200 pounds per square inch in beams 13.1

feet long having a cross-section about 8x16 inches and reinforced near one of the smaller sides by 4 metal rods of $\frac{7}{8}$-inch diameter placed 1.3 inches from the face. The latter specimens were prepared to be tested for bending.

It is superfluous to point out the importance of the above statement as to the magnitude of the interior stresses in members of the usual mixtures and of dimensions similar to those met in practice. Neglecting this kind of stresses, some engineers have made grave mistakes in the interpretation of bending experiments and have established incorrect formulas and rules, especially on the subject of stresses in compression members. They have assumed that if a certain specimen has undergone a shortening, i, its reinforcing bars, which had a coefficient of elasticity E, were compressed to a stress $E\,i$, neglecting the addition which has to be made to the latter stress for the shrinking of the concrete, if it has hardened in air, and which usually exceeds it in amount. The above considerations are sufficient to compute the stresses in compression members as long as the elastic limits have not been surpassed, neither in the concrete nor in the metal; but this is only one side of the question.

Without entering into a discussion of the unit stresses which may be allowed for the various elements of concrete-steel structures, it is evident that the basis of any computation must be the knowledge of the stresses which are induced in these elements at the instant at which, for the first time, there appears any danger for the one or the other of them. It is, therefore, important to know the stress caused by the reinforcing steel in a member in compression at the instant where it begins to fail by the crushing of the concrete, which takes place a long time before that of the steel.

A concrete of common quality can stand without crushing a reduction in length of 0.07 to 0.10 per cent. and sometimes more. Such a compression will cause a stress in the metal of 20,000 to 29,000 pounds per square inch, if

the coefficient of elasticity be 29,000,000 pounds. This stress added to the previous stress of 7,000 to 14,000 pounds, gives a total of 27,000 to 43,000 pounds per square inch of the metal, which is equal and even superior to the elastic limit of the iron and mild steel which is usually employed. Therefore, before the crushing of the concrete, the reinforcing bars are almost always stressed up to their elastic limit, unless the elastic limit of the bars be exceptionally high or the concrete exceptionally poor.

This stress cannot be appreciably surpassed because a very great decrease takes place in the value of the coefficient of elasticity of the metal as soon as the elastic limit has been exceeded, and the stresses increase, therefore, with an extreme slowness which is limited by the small deformations which the concrete can still undergo without crushing.

Hence, it may be stated that *in concrete members reinforced by longitudinal rods connected by cross-pieces or ties too weak or too far apart to bind the concrete sufficiently together crosswise, the total resistance to crushing varies little from the sum of the resistances offered by the crushing strength of the concrete and the longitudinal bars when stressed up to their elastic limit. During the elastic period the metal, which has been compressed before by the shrinking tendency of the concrete, causes important stresses.*

2. Concrete Reinforced by Transverse Rods.

Whatever the mode of rupture of concrete in compression, the crushing of the same must be retarded by the use of reinforcing rods put in planes perpendicular to the direction of the external pressure and sufficiently near to each other. The tendency to slide along oblique planes is, indeed, resisted by reinforcing bars which cut these planes, whether parallel or perpendicular to the direction of the pressure. Rupturing along surfaces parallel to the pressure is directly opposed by transverse reinforcing.

The idea of using transverse reinforcing is not new, and, while it may be still older, it is sufficient to mention that it was experimented upon in 1892 by Koenen and Wayss. Since then Harel de la Noé has theoretically explained the advantages of transverse reinforcing and has made and inspired some very interesting applications. The transverse reinforcing may consist of a series of rods placed on diameters, all passing through the center of the section, or of a net with rectangular openings, or of circumferential rods which constitute hoops embedded in the concrete to a depth required to protect the metal from the action of atmospheric influences.

The author has not made any experiments on the first system which concentrates the metal around the center where it is the least useful. He has limited his preliminary experiments to reinforcing consisting either of circumferential hoops or of netting wires at right angles and parallel to the sides of the section. For equal weights of metal the resistance to crushing was appreciably more than twice as great for the circumferential reinforcing as for the wire netting.

Without entering into a theoretical discussion, the above result can be explained by a simple observation. If the external layers of a prism reinforced by rectangular wire nets are considered the lateral thrust outwards to which they are subjected by the pressure at their base will in nowise be resisted by the rods parallel to these layers or faces, and nothing prevents them from separating from the central mass simultaneously with the concrete in which they are embedded. Of course, the bars at right angles to the faces considered offer a resistance to the outward thrust, but only to such extent as they adhere to the concrete. This adhesion is proportional to the area of contact, and is zero at the ends and only increases in intensity as the distance along the bars increases from the faces, but these faces are just the layers most exposed to crushing.

To remedy this fault the author has first employed iron rods so connected as to support each other, and then nets of wires interwoven in a manner which promised the best results. After all these arrangements the crushing beginning at the face has gradually spread toward the center and it became apparent why, for equal weights of steel, not more than one-half of the resistance shown by the hooped concrete was obtained. It was as a result of the above experiments that all further investigations were directed to concrete reinforced by hoop-like rods.

3. Theoretical Considerations on the Resistance of Hooped Concrete.

The inner forces acting in solid bodies are often placed in two different classes. The name, cohesion, is generally given to the inter-molecular action, and it is known that it varies in proportion to the distances of the molecules from each other up to a certain point which is called the "elastic limit." As a premise nothing is supposed to be known of the effects produced by cohesion above the elastic limit; but at the same time it is generally admitted that friction exerts an action in the interior of bodies similar to that exerted on their surface.

However, this division, which may appear arbitrary, is not generally accepted and the deductions made from it may be disputed. We will leave the purely theoretical considerations and will attempt to attain the practical aim of the engineer, which is to formulate rules which will enable him to predict the mechanical properties of the materials. The following method was adopted for investigation:

A certain number of prisms of concrete of different qualities and surrounded by hoops of various arrangements and sizes was prepared. Some had, also, longitudinal reinforcing rods. These prisms were submitted to increasing pressures and the shortenings produced were

measured together with the loads. By a well-known formula for the thrust of a granular mass, the resistance was computed which would be offered by a prism of the same dimensions, reinforced in the same way if sand without cohesion were put in place of the concrete. The same coefficient of friction was assumed and the same percentage of swelling of cross-section to decrease of length. This was computed for each observed deformation. It is evident that the excess of the observed resistance of a concrete prism over the similar resistance of sand corresponding to the same deformation can only be attributed to the direct or indirect effects of the cohesion of the concrete. Without entering into a discussion on the character of this difference in resistance and without attributing to the name a precise scientific meaning, we shall call this excess the "specific resistance of the concrete."

From this definition it follows that to determine the total compressive resistance of a hooped concrete prism it will suffice to add the specific resistance of the concrete to the resistance of a prism of sand having the same hooping and the same coefficients of friction and transverse swelling. The latter resistance can be computed. To make use of this arbitrary distinction it must be possible to predict the specific resistance of the concrete in hooped members from the resistance of concrete of the same quality not hooped. It will be seen that this can be done as far as is required.

4. The Resistance of Hooped Sand.

Of course, sand without cohesion cannot be actually hooped otherwise than by a continuous shell of the same weight, as the hooping rings of concrete. This difference is of no importance as far as the following considerations are concerned. The crushing resistance which the hooping will give to sand without cohesion is easily computed by any formula for the thrust of a granular mass.

If p represents the pressure per square inch which is exerted on the upper end of a vertical cylinder of a non-cohesive material, the angle of friction of which is f, and the weight of which can be neglected relatively to the external pressure, it is known that to prevent crushing a pressure must be applied on the sides equal to $\frac{p}{K}$ per square inch, where $K = tan^2 \frac{1}{\frac{f}{2}}$. By the aid of this formula the effect of hooping can be easily computed for a cylinder. In fact, if A be the area of each of the two symmetrical sections which a meridianal plane cuts in the hooping; if, further, r be the radius of the base of the cylinder and h its height, the pressure per unit area of surface of contact which the hooping exerts on the sand will be equal to $\frac{A}{r\,h}$, for each unit of tensile stress in the metal. From the formula for K it follows that the upper base of the cylinder will sustain $\frac{K\,A}{r\,h}$ per unit of area and $\frac{\pi\,K\,A\,r}{h}$ for the whole area of the base, πr^2.

The volume of the hooping metal being $2\ \pi\ \ r\,A$, the ratio u of the resistance given by the hooping to the sand to the volume of the metal employed will be $u = \frac{K}{2\,h}$. It is evident that the corresponding ratio u_1, for the longitudinal reinforcing members which sustain the pressure directly as they are generally employed in reinforced concrete construction, will be $u_1 = \frac{A}{A\,h} = \frac{1}{h}$. We thus get $\frac{u}{u_1} = \frac{K}{2}$, and experiments have given $K = 4.8$ for sand and appear to lead to the same value for concrete. It thus follows that *the resistance given to sand by the hooping is* 2.4 *times greater than the direct resist-*

ance of the longitudinal reinforcing members of same weight when the tensile stress in the former is equal to the compressive stress in the latter.

Thus 2.4 is also the ratio of the crushing resistances of the two types of reinforcing for equal weights of reinforcing metal. This is so because crushing takes place in hooped members as well as in members longitudinally reinforced when the elastic limit of the metal has been reached, which is the same for tension as for compression. In fact, the hoops yield then too rapidly to prevent swelling.

Crushing is not the only danger which has to be taken care of in compressed members; they can also fail by flexure, as columns, and then their resistance is proportional to their coefficient of elasticity, that is, to the ratio of the unit pressure they sustain to the shortening which they undergo. If m represents the ratio of the transverse swelling to the longitudinal shortening the hoops will elongate by $m\,i$ only when the longitudinal members will be shortened by i. The tensile stresses in the former and the compressive in the latter are thus proportional to these deformations, mi and i, and, hence, the corresponding increases in the coefficients of elasticity due to the hooping or longitudinal reinforcing are to each other as $\frac{miK}{2}$ to i, or as $\frac{m\ K}{2}:1$, which gives the numerical value of $2.4\ m : 1$.

It is, of course, out of the question to use hooped sand, and the preceding considerations are of interest only for the indications which can be deduced from them as to the resistance of solids. The value m, which has been mentioned in the above, refers to hooped concrete and no conclusive experiments have been made for its determination. The only experiment which can be mentioned referred to unreinforced concrete and gave for m the value of 0.4. It is evident that smaller values will be found

for hooped concrete, and they will be the smaller the greater the percentage of metal, because the hoops resist the swelling, increase the density, and make the molecules approach each other, which is the cause of the increase in the coefficient of elasticity.

The author has adopted 0.375 for the value of m because this value gives resistances which, on the average, agree sufficiently with the results of experiments made on prisms of hooped concrete. If noticeable difference occur, they are on the safe side. There is evidently no reason why this value of m should also be taken for sand. The following conclusion may thus be made without raising objection.

If there existed a noncohesive material for which K and m had the values 4.8 *and* 0.375, *which appear to agree with concrete, the friction caused by the hooping would give it a coefficient of elasticity which would be* $0.375 \times 2.4 = 0.90$ *of that of longitudinal reinforcing members of the same weight as the hooping.*

The modifications which cohesion will cause in the effects of friction cannot be easily predicted, and the conclusions drawn as to noncohesive bodies are of interest only in so far as they permit, in the absence of an exact theory of hooped concrete, the deduction of sufficient rules for the computation of its resistance and coefficient of elasticity.

5. Experimental Researches.

The above considerations referred to bodies without cohesion; and it is important to determine whether in concrete the effects of cohesion act in addition to friction and in what way. Experiments were made to verify the agreement of the resistance of a sand cylinder in a shell with the formula for earth thrust. It was thus that the value 4.8 for K was obtained, which agrees with the angle of friction. A series of experiments was then made at Quimper, in 1901, on small prisms of mortar 1.575

inches in diameter hooped by a fine iron wire. The deformations were not measured and the results can, therefore, only be used to verify the resistance to crushing and to compare the same to the assumptions made above. The accompanying table shows the results obtained from some of the prisms.

The figures given in the table are significant. The iron wire employed for the hooping was drawn cold and did not have a definite elastic limit. From its curve of deformation, 78,200 pounds per square inch appeared to be the value, after passing which the stress in the metal increased too slowly for its elongation to be able to maintain the real efficiency of the reinforcing. This value of 78,200 pounds per square inch has been multiplied in the table by the ratio of iron to concrete, giving the values in the next to last line, representing the compressive resistance which would have been offered by the metal if it were used as longitudinal reinforcing bars instead of hooping wires. The ratios obtained in the last line of the table thus give the coefficient of efficiency from the point of view of the compressive resistance of metal employed in longitudinal reinforcing or in hooping. For sand, as seen in section 4, this ratio is 2.4 and the figures of the last line show that for mortar this ratio has not deviated far in these experiments.

The compressive values obtained should be noted. Of the prisms one was of mortar which had had time enough to set. With a volume of metal equal to 0.034 of the total volume and without any longitudinal reinforcing it showed a resistance of 10,500 pounds per square inch of total section. It will be useful to compare the resistance of this prism with the resistance of an iron prism of the same weight. The hooped concrete had a density of 2.4, and that of iron is 7.8. To compute the pressure which an iron bar of the same weight will get per square inch of section, 10,500 must be multiplied by 3.2, the ratio of

the densities of the two materials, giving 33,600 pounds per square inch. Since not more than 36,000 to 39,000 pounds per square inch can be expected of the total area of a riveted iron section weakened by numerous holes, it may be said that a prism of an ordinary mixture, reinforced by an average percentage of hoops, has shown a compressive resistance in the neighborhood of that of ordinary iron.

TABLE XIII.

Weight of cement per cubic yard of sand	675 Pounds				730 Pounds
Age of mortar tested, days	8	14	22	23	100
Ratio of volume of iron to volume of concrete	0.02	0.03	0.04	0.02	0.034
Resistance to crushing in pounds per square inch of total section	4,870	6,540	7,360	4,930	10,500
Resistance to crushing of concrete not reinforced	569	711	853	853	2,420
Increase of resistance due to hooping	4,301	5,829	6,507	4,077	8,080
Product of ratio of iron to concrete by 78,200 pounds	1,564	2,346	3,128	1,564	2,658
Ratio of the values of the last two lines	2.7	2.5	2.1	2.6	3.0

It will be seen in what follows that the results of hooping are less advantageous for the coefficient of elasticity, and, therefore, for the resistance to flexure as a column than for that of the crushing resistance. To study this question experiments had to be made on long members and their deformations measured. With the aid of M. Hennebique, 38 prisms of octagonal section of 5.9 inches diameter were made. Different reinforcing was embedded, and the concrete consisted of the usual mixture of 420 pounds of Portland cement to the cubic yard of sand and gravel in the proportion of 1 : 2 in some prisms and double the amount of cement in others. Some prisms had a length of 1.64 feet and were especially intended to test the crushing resistance; the others had a length of 4.25 feet and were used to study the elasticity and the

ductility of hooped concrete, which is one of its characteristic properties and one of the most important for safety.

Each specimen teaches its lesson, but it is plainly impossible to describe all the results obtained, comprising about 1,200 observations of deformations. They have, therefore, been grouped so as to throw most light on the important points.

The first group consists of six prisms. Prism A was not reinforced. It crushed under a load of 1,050 pounds per square inch. Prism B was reinforced with helicoidal spirals of 5.5 inches average diameter made of cold-drawn $\frac{1}{4}$-inch iron wire spaced 1.18 inches centers to centers. It crushed under a pressure of 5,120 pounds per square inch of total section. Prism C was reinforced by helicoidal spirals of 5.5 inches average diameter made of cold-drawn iron wire of 0.17 inch diameter and spaced 0.59 inch centers to centers. Without crushing it stood a pressure of 5,400 pounds per square inch, which was the highest pressure supplied by the testing machine employed. Prisms D and E were reinforced respectively the same as B and C, and in addition by 8 longitudinal wires $\frac{1}{4}$ inch in diameter leaning against the inside of the spirals. They failed as columns before crushing, under pressures of 4,550 to 4,700 pounds per square inch. Prism F had 8 longitudinal reinforcing wires 0.35 inch diameter tied together by belts of iron wire 0.17 inch diameter spaced 3.15 inches apart, that is, closer than they are usually in concrete steel constructions. It failed under a pressure of 2,420 pounds per square inch. The following phenomena were noticed without the aid of measuring devices.

6. General Properties of Reinforced Concrete and Hooped Concrete.

The unreinforced prism A broke suddenly, without any signs of approaching danger. The failure of F was almost

as sudden. Its breaking load did not exceed by more than 7 per cent. the load producing the first cracks. The reinforcing rods bent outwards between their cross-connecting ties and the concrete crushed. Concrete in compression when not reinforced or when reinforced by longitudinal

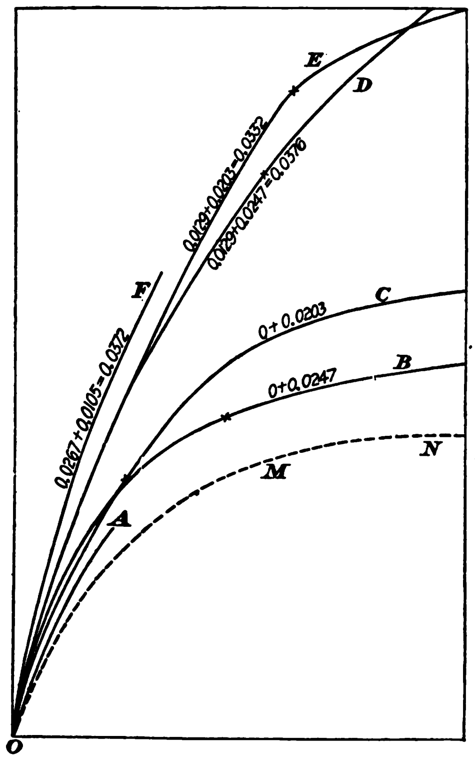

FIG. 19.

rods must be classified among the materials which break suddenly without much deformation. Quite different is the behavior of hooped concrete, as will be seen.

The hooped prisms behaved at the beginning the same as the others and showed only very small deformations un-

der small loads; but this so-called elastic period did not end with the failure of the specimen. The shortening was observed to increase rapidly and cracks appeared in the concrete covering the hoops, first fine, then more and more pronounced. The curves of deformation of these prisms are represented in Fig. 19. The cross-marks in them indicate the appearance of cracks in the prisms B, D, E. These superficial injuries appeared in prism C only after the considerable shortening of 0.355 per cent.

The apparatus used did not measure exactly the shortenings above the figures given in Table XIV, but rough measurements showed that failure took place after deformations greater than 3 per cent. of the lengths. Table XIV gives the shortenings observed under the pressures indicated in the first line.

TABLE XIV.

Shortenings of Reinforced Concrete Prisms, in One-Thousandth Parts of Length.

Pressures in Pounds per Square Inch.	108	250	391	545	696	839	992	1,145	1,288	0	1,288	1,667	2,043	2,420	2,795	3,175	4,240
Prism A*...	0.05	0.12	0.17	0.25	0 31	0.40	0.63	......	......	......	......	......	......	......		...	......
Prism B†.............	0,00	0.02	0.06	0.17	0.24	0.30	0.39	0.54	0.67	0.07	0.71	1.37	4.39	8.39	12.94	14.48	......
Prism C‡.	0.01	0.015	......	0.17	0.25	0.37	0.46	0.62	0.68	0.15	0.61	1.08	1.58	3.55	5.33	6.07	8.14

* Rupture at 1,050 pounds. † Rupturc at 5,120 pounds. ‡ Sustains 5,400 pounds.

TABLE XIV — *Concluded.*

P. essures in Pounds per Square Inch.	483	968	1,449	1,936	2,415	2,898	3,381	3,864	0	1,936	3,874	4,347
Prism D*......................	0.19	0.33	0.59	0 90	1.26	1.67	2.26	3.10	......	......	......	....
Prism E†	0 11	0,33	0.60	0.85	1.18	1.55	1.93	0	0.48	1.63	3.18	4 44
l rism F‡.....................	0 15	0.33	0.44	0.74	1.03	......	......	......	......	......	......	....

* Failed by column pressure at 4,550 pounds. † Failed by column pressure at 4,700 pounds. ‡ Rupture at 2,420 pounds.

The prisms A, B, C, were 35.4 inches long and the prisms D, E, F, 51.2 inches. On each of the curves of deformation shown in Fig. 19 are inscribed two figures and their sum. The first of these figures gives the proportion of the area of the longitudinal reinforcing members to that of the concrete, and the second that of the hoops, or, in their absence, of the iron ties. The third figure gives the total proportion of metal. By comparing the percentages of the iron to the concrete with the corresponding crushing resistances, the considerable superiority of hooped concrete, from this point of view, becomes evident. The table giving the results of the Quimper experiments shows this very clearly.

In concluding it may be added that the first examination of the results of this group of experiments has shown at once *that while concrete not reinforced or reinforced by longitudinal rods, even when tied together by ties appreciably nearer to each other than is usually the case, breaks in compression under small deformations and with out any notice, hooped concrete sustains, without crushing, considerably heavier loads and only fails a long time after cracks in the faces and exaggerated deformations have called attention to the approaching danger.*

7. The Spacing of the Hoops.

Examining the curves of Fig. 19 more closely, an apparent anomaly is noticed. The ordinates which correspond to a given abscissa, or, in other words, the pressures sustained by the differently hooped prisms for the same shortening, have no relation to the proportion of metal in the hoops or in the longitudinal reinforcing members. The explanation of this fact has been furnished by the observation of the conditions of the tests.

While testing prism B, cracks appeared under the light load of 1,730 pounds per square inch, and soon after the

concrete began to chip off and finally failed between the spirals which served as hooping. These spirals were 1.18 inches apart. The failure of the prisms, which took place under a pressure of 5,120 pounds per square inch was due to the failure of the concrete, and there was nothing to indicate that the metal had reached its elastic limit. In prism C, whose spirals were only 0.59 inch apart, the cracks did not show before a pressure of 2,480 pounds per square inch was exerted. Chipping occurred also later than in B, and under a pressure of 5,400 pounds no failure of the concrete was observed, and the prism did not fail. In D and E cracks appeared under pressure of 2,900 to 3,360 pounds per square inch. These prisms contained, besides the same spirals as B and C, longitudinal rods, which were in contact with the interior surfaces of the spirals, and formed with them a network which efficiently resisted the lateral failure of the concrete. Computations prove that the superiority of D and E, in this respect, considerably surpasses the resistance which could have been offered by the longitudinal rods alone. Connecting these facts to the figures given in Table XIII, and to the observations made on other series of experiments, certain conclusions are arrived at as to the results obtained on the subject of the spacing of hoops and longitudinal reinforcing rods.

When the spacing of the spirals did not exceed one-fifth the diameter of the prisms, resistances were obtained almost independent of this spacing. On the contrary as far as it concerned the appearance of cracks in the external layer, the elasticity, and, hence, also the column resistance, the results were the more successful the nearer the hoops were to each other. The resistance of these prisms was still much more increased by the addition of longitudinal rods leaning against the interior surface of the hoops. *The facts mentioned and others which have been observed lead to the adoption of a spacing of the spirals of one-sev-*

enth to one-tenth the diameter when longitudinal reinforcing rods are added.

The question may be raised whether it is proper to admit that the effects of hooping are the same, whatever the absolute values of the dimensions, as long as the relative values of these dimensions to the diameter of the post remain the same. Experiments on widely different dimensions have proved it to be so, and the following consideration will explain it: If, taking the simplest case, the spirals are replaced by continuous hoops extending between planes perpendicular to the axis, the deformation which the continuity of hooping will indirectly prevent will consist of the swelling of the little cylinders inclosed between the tangent planes of the spirals. If the end pressure which is exerted on two similar prisms of different diameters has the same unit value, the lateral pressure required to prevent swelling will, in all probability, also have the same unit value. Such is certainly the case with sand. The total value of the lateral pressure will, for the small cylinders considered, be proportional to their sections along a diametral plane which is equal to the product of their height by the diameter; that is, to the square of the diameters since the prisms are similar. The hoops will offer this resistance to swelling, but it can only be transmitted from the metal to the small cylinders by the friction and cohesion acting on their bases whose areas are also proportional to the square of the diameter.

If two cylinders of different diameters are considered, all dimensions of which are proportional to each other, the shearing stress which will be produced in the planes tangent to the hoops will have the same value when the unit pressure on the prisms are the same. In other words, prisms of proportional dimensions behave the same way under the same unit pressures, and it seems to be justifiable to express the rules as to the dimensions of the hooping element as a function of the ratio of these dimensions to the diameter, as was done above.

8. The Ductility of Hooped Concrete.

A metal tube $7\frac{1}{2}$ inches in diameter was filled with Portland cement and bent to a radius of curvature of the neutral line of 21.6 inches. The metal shell was then removed and a piece of the cement was cut out. The cement which underwent such great deformations did not break, and on the compressed side only infrequent cracks were observed. Tests proved that it still had a great resistance. This observation on concrete inclosed in a metal shell led to the belief that similar results would be obtained for hooped concrete. Numerous experiments have indeed proved that in prisms which had bent under heavy pressures the concrete did not break and it maintained its cohesion. A hooped prism of the proportion of 840 pounds of cement per cubic yard of gravel and sand, which was subjected to a pressure of 7,940 pounds per square inch of original section showed great deformation, bending in the shape of the letter S with a greatest versed sine of 0.4 inch in a length of 13 inches. The curvature was much sharper at the middle so that the least radius of curvature was about 2 feet. The portion of concrete nearest to the outside did not show any transverse cracks, and hence could not have elongated much. The flexure of the prism was, therefore, produced almost wholly by the shortening of the opposite fibers, for which computation gave the enormous figure of 17 per cent.

TABLE XV.

Effects of Repeated Longitudinal Compression on Four Hooped Concrete Prisms.

First Loading and Unloading.

Pressure	1,053	1,850	2,375	3,330	3,825	4,490	3,825	3,300	2,375	1,850	1,053	0	....	....
Shortening of G	.0047	.0095	.0130	.0158	.0260	.0480	.0480	.0378	....		....	.0146	....	....
Shortening of H	.0173	.0276	.0350	.0437	.0567	.0709	.06"8	.0615	....	.0496	.0480	.0248	....	....
Shortening of I	.0118	.0205	.0264	.0350	.0437	.0449	.0445	.0400	.0354	.0335	....	.0102	....	....

Second Loading.

Pressure	1,850	3,300	4,490	4,890	5,150	5,420	5,600	6,210	6,740	
Shortening of G	....	.0449	.0540	.0645	.0662	.0685	.0827	.1356	.1940	Flexure at .1980
Shortening of H	.0540	.0642	.0745	.0772	.0984	.0945	....	....	....	Flexure at .1760
Shortening of I	.0335	.0406	.0540	.0583	.0680	.0760	.0875	....	....	Flexure at .2195

First Loading and Unloading.

Pressure	526	1,053	1,990	2,375	2,815	3,300	3,565	3,300'	3,040	2,815	2,375	1,990	1,053	526
Shortening of J	.0146	.0236	.0276	.0323	.0437	.0567	.0831	.0827	.0807	.0787	0.768	.0748	.0615	.0540

Second Loading.

Pressure	0	526	1,053	1,990	2,815	3,040	3,300	3,570	3,980	4,350	4,750	5,150	5,285	....
Shortening of J	.0350	.0524	.0599	.0717	.0827	0.831	.0842	.0905	.0945	.0984	.1260	.1693	.2047	....

NOTE.—Pressures are given in pounds per square inch, and shortenings in decimals of an inch.

It is not certain whether the concrete could have stood this deformation if the sharpest curvature had not extended over only a short length; the obliquity and warping of the cross-sections could have had an important influence on the curvature. But the fact remains that the above prism stood considerable deformation. The hooping spirals and longitudinals were afterward removed from this prism and the remaining concrete was coherent enough for its whole length of 4.25 feet to be handled without breaking. It was put on two blocks 3.61 feet apart and it required 55 pounds to break it by bending. One of the halves of the prism which was less bent by the warping, but which stood the same average pressure of 7,940 pounds was put on two supports 20.5 inches apart and a load of 428 pounds was required to break it.

The tensile resistance indicated by this bending test figures, by the formula $R = \frac{10\,M}{d^3}$, 205 pounds per square inch, in which M represents the bending moment and d the diameter, which the removal of the reinforcing had reduced to $4\frac{3}{4}$ inches. This tensile resistance of 205 pounds does not much differ from the initial tensile resistance of the concrete. In the compression tests without column flexure not more than 3 per cent. of shortening was observed before failure. The difference of these results is due to the considerable swelling which, in this case, is required by such shortening as would lead to failure, outside the elastic limit.

In two other prisms the compressive resistance instead of the resistance to bending was tested after the removal of the reinforcing hoops and longitudinals. The first, f the same proportions as the one above, bent under a essure of 6,970 pounds per square inch, which caused shortening of 0.6 per cent. The average compressive esistance of the plain concrete was 1,420 pounds, and its eal resistance must have much exceeded this figure, as the pressure was not well applied at the ends. A similar

test was more carefully made on a prism mixed in the proportion of 630 pounds cement per cubic yard, and which stood a pressure of 10,270 pounds per square inch with a shortening of 2.4 per cent. on the average and 2.8 per cent. on the most stressed side. After removal of the reinforcing spirals, the inside cylinder, which had about 10.5 square inches section, sustained a pressure of 9,700 pounds.

From the above tests, selected as they are from many similar tests, it must be concluded that *hooped concrete sustains without disintegrating considerable shortening and conserves a great part of its original resistance; and that with the small deformations which occur in structures, the resistance of hooped concrete can be considered to be constant from the instant when it reaches its maximum.* The analogy between this phenomenon and the behavior of concrete in tension is evident.

9. The Elastic Behavior of Hooped Concrete. Experimental Data.

The amount of deformations of a structure, which is, in certain cases, of importance, as well as the column resistance, which is of prime importance for all columns, both depend on the coefficient of elasticity. It is, therefore, absolutely necessary to investigate carefully the elastic behavior of hooped concrete.

A. The Elastic Behavior under a First Load.

From among a great number of results obtained from tests of prisms those given by four octagonal prisms 5.9 inches in diameter and 4.27 feet long have been chosen for the interpretation of the results. These prisms were reinforced by helicoidal spirals and longitudinal rods, as follows:

Prisms G and H, 840 pounds cement per cubic yard, spirals $\frac{1}{4}$-inch wire spaced 0.79 inch, 8 rods, 5/16 inch

diameter; Prism I, concrete and spirals same as above, 20 rods, 0.276 inch diameter; Prism J, 420 pounds cement per cubic yard, spirals same as above, 8 rods, 0.276 inch diameter. Table XV shows the shortening of these prisms under the given loads, the loading being repeated as indicated.

In all of these specimens the spirals had the same dimensions, and in the first three the concrete was of the same mixture. Similar results might have been expected of them, but the facts proved otherwise. While for the first prism the pressures below 2,845 pounds per square inch gave a coefficient of elasticity of 7,111,000 pounds, the second of identical composition gave only 2,845,000. This great difference was due to the quantity of water used for the mixing of the concrete; it was correct for the first and excessive for the second, and gave a soft concrete which crumbled under the hammer and could not acquire the required compactness for a high coefficient of elasticity. The first lesson taught by these experiments is the great irregularity of concrete as usually fabricated and not carefully inspected, and the doubtful value of results based on the coefficient of elasticity.

If the general behavior of the deformation curves of the prisms be studied, it is noticed that they show an important change of inclination after a certain pressure has been exceeded, the same as is shown for ductile metals. This point may be called the elastic limit, but without attributing to it the sense usually given to this limit. An elastic limit, in the proper meaning of the words, does not exist for concrete. Prisms of the same proportions do not show so much irregularity in their elastic limit as in their coefficient. The first three specimens, G, H, I, thus had an elastic limit varying between 4,830 and 5,400 pounds per square inch, while the coefficient of elasticity varied between 2,850,000 and 7,100,000 pounds. The elastic limit, moreover, varies with the proportion of

cement, while the coefficient of elasticity, on the contrary, is little influenced by it. For prism J it reached only 2,845 pounds per square inch. These results are in accord with what is known of concrete that is not reinforced.

B. The Elastic Behavior under Repeated Loads.

The observation of the deformations during the unloading and reloading of the specimens, remaining always below the first load, has given results of great practical interest. The first thing shown, which could have been predicted, was a permanent shortening which increases if the same load is repeated, but more and more slowly, and tends rapidly toward a final limit. A reduction of the final deformations is thus obtained and with it an appreciable increase in the coefficient of elasticity for the succeeding unloadings and reloadings of the specimens. The second result, which is more important, could not have been foreseen. It is the form of the deformation curves and especially the direction of their curvature, which is concave to the pressure axis, while during the first application of load it curves in the inverse sense. It follows that *the coefficient of elasticity which is represented by the inclination of the tangent to the curve of deformation increases with the pressure in the unloading and reloading instead of decreasing with increasing pressure as under the first application of load.* The importance of this fact will be more fully discussed under "Column Resistance," in section 13; but it is well to indicate here its bearing.

Evidently flexure is to be feared in a column under high pressures and it is, therefore, unfortunate that the coefficient of elasticity, which is directly proportional to the column resistance, decreases with the increase of pressure. Such is the case under the first application of the load for hooped or otherwise reinforced concrete and

also for structural iron and steel. On the other hand, it must be considered especially fortunate that hooped concrete which has been subjected to a first load has a coefficient of elasticity which is the greater the higher the pressure becomes, provided it does not exceed the first load. This fact has, to the author's knowledge, never before been observed on other materials. To better discuss this point the results of a very exact experiment, made at the laboratory of the Ponts et Chaussées, are given in Table XVI. The prism tested had an octagonal section of 4.3 inches diameter and was made of concrete of the proportion of 1,000 pounds of cement to 32 cubic feet of gravel 0.2 to 1 inch in diameter and 10.7 cubic feet of sand passed through a screen of 0.2-inch holes. The helicoidal spirals were of iron wire 0.17 inch in diameter and 0.82 inch centers to centers. The average diameter of their generating cylinder was 3.76 inches. In addition the prism was reinforced by 8 longitudinal wires of the same size and material. The length of the prism was 51.18 inches.

TABLE XVI.

Effects of Repeated Longitudinal Compression on a Hooped Concrete Prism.

First Loading and Unloading.

Pressure in pounds per square inch....	128	441	810	1,180	1,620	1,990	2,360	1,620	1,180	810	441	128	....
Shortening in one-thousandth parts...	.0	.12	.36	.52	.76	1.16	1.60	1.54	1.52	1.34	1 00	0.76	....

Second Loading and Unloading.

Pressure	1,180	1,620	1,990	2,360	3,170	3,990	4,720	5,530	5,160	4.720	3.170	1,620	128
Shortening	1.28	1.38	1.56	1.76	2.76	3.96	5.68	9.14	9.12	8 60	8.24	7.56	6.42

Third Loading and Unloading.

Pressure	1,620	3,170	4,720	5,530	6,340	7,525	7,910	7,525	7,100	6,340	4.720	1,620	128
Shortening	7.08	7.88	8.64	9.08	10.1	13.9	16.72	16.68	16.7	16.6	16.24	14.8	13.9

Fourth Loading and Unloading.

Pressure..............	1,620	4,720	6,340	7.100	7,525	7,910	8,710	10,290	9,280	7,910	6,340	128	....
Shortening	14.04	15.6	16.2	16.7	17.0	17.86	19.5	......	24.1	24.1	23.7	20.16	....

Column flexure prevented the measurement of the shortening under the pressure of 10,290 pounds per square inch. Fig. 20 shows the plotted results of these and numerous other experiments not given in Table XVI.

The concrete surrounding the spirals was removed to a cylinder of 3.37 inches, which just passed through the

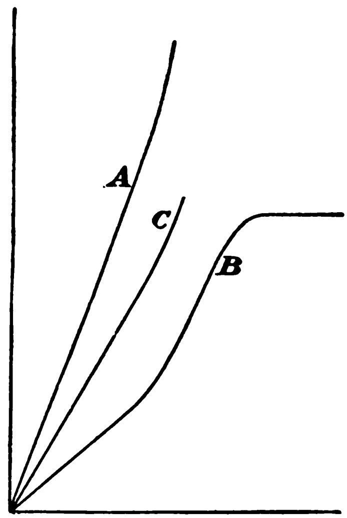

FIG. 20.

middle of the reinforcing wires. The effect of the external layer of concrete was thus eliminated because of the difficulty in determining its action precisely. The pressure was successively raised in four applications to 2,360, 5,530, 7,910 and 10,290 pounds. After each of the highest pressures, the load was gradually decreased and then applied again and the shortening measured carefully under each pressure.

The coefficients of elasticity corresponding to the different pressures are proportional to the tangents of the

deformation curves. They are given below as determined under the first reloading and under the following unloadings and reloadings. For simplicity's sake only the average of the two last operations is given:

Pressures in pounds per square inch.	853	1,990	4,270	6,830
Coefficient of elasticity under first loading	2,180,000	853,000	384,000	284,000
Coefficient of elasticity under unloadings and reloadings.	1,990,000	4,910,000	2,420,000	3,275,000

After the considerable pressure of 10,290 pounds per square inch had been applied it was taken off and it was found that the prism had gone back to a coefficient of elasticity as high as after the application of the lightest pressures. The experiments given in Table XV lead to the same conclusions. Besides, the comparison of the relative values of the different prisms shows that the first application of load improves the concrete in the same degree as it had initially been deficient. It is natural, indeed, that a strong pressure should diminish the final deformability by bringing the particles nearer together, and that this effect should be greater the less the concrete has been tamped and the farther apart the particles were before.

Concluding the discussion of the results of the above experiments and of the effect of the first loading it may be stated:

**The application of a first pressure on a hooped prism, no matter how high the pressure may be, as long as it is below the breaking load, has the effect of raising its elastic limit up to that pressure. The coefficient of elasticity which is subsequently developed by the hooped concrete under all the variations of the pressures between the lowest and the previously applied load is higher than the highest coefficient of elasticity which the prism had before the test load and which held true for a low pressure only.*

The increase in the coefficient of elasticity of the concrete after the test load, as compared to the coefficient before, is so much the greater, the less the proportion of cement and the lower the quality of the concrete.

These properties are similar to those of the ferric metals whose elastic limit also increases to the pressure once applied; but they differ in the curvature of the deformation curve, which shows increasing values of the coefficient of elasticity of hooped concrete at pressures below the test load. Hooped concrete in compression does not show, in this regard, any similarity to reinforced concrete in tension, whose coefficient of elasticity decreases considerably after appreciable deformations, and the more so the greater the deformations have been.

It will be seen below that it appears possible to utilize the above properties of hooped concrete. This has never been attempted for the metals, and it is easily seen why. It is almost impossible to subject the individual members of a metal structure to a preliminary pressure. They do not, generally, offer convenient bearing places and it is to be feared that the riveted connections would be distorted under high stresses. But it seems, on the contrary, possible to submit to high pressures voussoirs of hooped concrete with plane faces. It is not certain that in some cases it would not be advantageous to obtain hooped prisms whose elastic limit would range from 7,000 to 10,000 pounds per square inch and whose stresses may run up higher still. To obtain such results the prisms will have to be subjected to pressures which will crush the concrete layer outside of the spirals. After the test a concrete coating will be put on in which asbestos may be substituted for the sand, which will make it much more ductile. It is already used in the making of roofing material of surprising solidity.

10. The Elastic Behavior of Hooped Concrete. Analysis of Facts.

The facts, as previously given, have only a limited value for practical purposes. It would suffice merely to state them if they were so numerous that from among them identical cases to those encountered could always be selected; but such a result is impossible, since the hooped prisms may vary almost indefinitely in the proportions of their solid elements, the proportion of the water used, the tamping, and the nature and dimensions of their longitudinal reinforcing. Aside, therefore, from the natural desire to solve the new problems, the practical needs compel us to investigate the laws governing the elasticity and resistance of hooped concrete. To find experimentally the effects of hooping to an absolute certainty it is evidently necessary to make identical prisms with and without hoops; but this is manifestly impossible. Two methods were adopted to avoid this difficulty. The first consisted in preparing specimens as nearly identical as possible and correcting for their differences in the initial values. The second seems to be exact, but can be used only for concrete tested by one load only. It consisted of testing a prism with its spirals and then testing the same prism after removal of the spirals.

For the application of the first method two prisms were made with the same care, of the same proportions and dimensions. They were first tested to establish their difference in elastic behavior. Then the exterior surface of both specimens was removed to the cylinder inclosed by the axes of the hooping wires and one of the prisms so prepared was again tested. The other prism was identically tested after removing the spirals. The results obtained for both were compared and the corrections required, as shown by the preliminary tests, were made. It was thus determined that the increase of the coefficient of elasticity due to the hooping was practically equal to 90 per cent.

of the coefficient of elasticity of the longitudinal wires of the same weight of metal.

The second method was applied to three prisms, of which two contained 500 pounds of cement and the third 1,000 pound per cubic yard of gravel and sand. Each of them was subjected to a maximum pressure repeated seven times, which was sufficient to reduce the increase of deformation between the two last loadings to a negligeable quantity, so that the concrete could be considered in a stable condition. The results obtained afterward were then comparable to each other. The deformations which were produced under the last load were measured and the coefficient of elasticity resulting from them for hooped concrete was computed. The external layer of the concrete, together with the spirals, was then removed and the prisms were again loaded. The new deformations were also measured and the corresponding coefficient of elasticity computed. If the removed outer layer of concrete could be neglected, the difference in the coefficients of elasticity could be wholly attributed to the hooping; but the error will be very small if the resistance of the removed shell of concrete be computed from the data obtained while testing the reduced prism.

Let n denote the ratio of the increase of the coefficient of elasticity due to hooping to the coefficient of elasticity of longitudinal rods of equal weight. The greatest shortening caused by the seven times repeated load was, in the three prisms, respectively, 0.074, 0.100, and 0.140 per cent., while n had the values 2.10, 1.60, 0.67. The error will thus be negligeable compared to the great differences between the values of n. In explanation of the above results it should be added that the shortening of 0.14 per cent. exceeded by 0.03 per cent. the shortening corresponding to the elastic limit, which was 0.11 per cent., that the shortening of 0.1 per cent. was not far from it, and that of 0.074 per cent was appreciably below. The

effectiveness of the elasticity of the hooped specimens was 210 per cent. of that of longitudinal rods, while the previously applied load was considerably below the elastic limit. It was reduced to 160 per cent. when the previous load approached the elastic limit, and it fell to 67 per cent. only of the elastic effect of longitudinal rods of same weight when the elastic limit had been previously exceeded. In practice the testing load will always be well below the elastic limit, except in unexpected cases, and the experiments, therefore, lead us to believe that in the loadings and unloadings after the test loads the hoops will produce elastic effect at least twice the value of those of longitudinal rods of the same weight.

At first sight these facts seem to disagree with those previously stated, which gave a value of 90 per cent. for n under the first load; that is, in the case where the elastic limit of the concrete has not been reached previously and has not even been approached. The explanation of this anomaly is furnished by the results obtained on the swelling of the compressed prisms in section 6. As long as the pressure exerted remained below 1,345 pounds per square inch, the measuring instruments did not indicate any appreciable swelling, but under higher pressures a rapid swelling was observed, which reached in two prisms 0.2 per cent. of the length of the specimen under a pressure of 3,130 pounds per square inch. The shortening of .07 per cent. which these prisms underwent under a pressure of 1,345 pounds certainly caused an appreciable swelling of the concrete, and it must be admitted that the spirals, whose diameters were measured, did not follow the surrounding concrete during the first movement. This is not surprising in view of the phenomena which take place during the setting and hardening of concrete in air.

It seems to be certain that the contracted concrete does not press effectively against the inner surface of the spirals before the load is applied, and that the first swelling only

causes a stronger contact between the concrete and the spirals. The hooping does not, therefore, produce its normal effect on the elastic behavior before the application of a certain load, which is in the neighborhood of 1,420 pounds per square inch. Such were the effects of hooping on the concrete during the first loading and it is easy to understand what they were afterward. Of the total deformation caused by the first load a permanent swelling remained, after unloading, sufficient to maintain an effective contact between the metal and the concrete, and the hoops showed their normal effect during all of the subsequent applications of pressure, unloading and reloading.

These considerations indicate an important difference between the action of hoops and longitudinal rods. In Section 1, it was seen that the rods are compressed by the shrinking of the concrete, that their effect is immediate, and that they reach rapidly the elastic limit of the metal. For steel this limit is reached when the load has caused an average shortening of 0.06 per cent. of the length. The hoops compressed by the shrinking must, on the contrary, first return to the state of molecular equilibrium before they take any tension, and this tension only becomes important when the longitudinal shortening has reached values above 0.06 per cent. of length. *The hoops only begin to be seriously stressed under the first application of load in prisms hardened in the air when the longitudinal rods having already reached their elastic limit are almost at their ultimate strength and cannot offer any further resistance.*

It is easy to understand why the action of the hoops extends through a wider range than that of the longitudinal rods. The elongation of the spirals is a result of the swelling of the concrete which is comparatively small and varies between 0.3 and 0.4, at the most, of the longitudinal shortening. The stress of the metal being proportional,

within the elastic limit, to its deformation, it is evident that having begun later it grows much more slowly in the hoops than in the longitudinal rods. It has been shown, however, by numerous experiments that, while stressing the metal but little, the hoops produce a considerable useful effect. These considerations explain the very great deformations which hooped concrete can undergo without sustaining injury either to the metal or to the concrete.

Quite a different aspect would be presented by concrete gradually hardened in water. Instead of contracting, the concrete will expand, causing the tensile stresses in the hoops and the longitudinal rods. The first will be stressed still higher by the application of load, while the second will first overcome their initial tension before taking compression.

It was seen that a first test load causes the hoops in concrete hardened in air to act effectively when loaded afterward, giving thus to hooped concrete a higher coefficient of elasticity. It is probable that similar results would be obtained without previous loading by keeping the hooped pieces in water or moist air during a sufficient length of time before finally exposing them to the air. Numerous experiences show, indeed, that the shrinkage of concrete exposed too soon to the dry air and the sun causes inconveniences. Cracks are sometimes noticed, which while of small importance are not ornamental, and the minute analysis of experiments on deformations has led the author to believe that the resistance and especially the coefficient of elasticity has somewhat decreased. It is certain that these inconveniences will be much reduced and possibly completely avoided by keeping the reinforced concrete in water or in moist places during several weeks before exposing it to the air and the sun. No doubt the change of humidity will also cause some shrinking, but it will be less and, what is very important, it will act on a body which has acquired much more strength and

which will not crack. Instead of acting on a raw concrete, it will cause internal stresses whose effects can be computed and which will not decrease the elasticity, because within the elastic limit, a decrease in tension causes exactly the same effect as an increase in compression.

11. The Elasticity and Resistance of the Concrete in Hooped Members.

To determine the effect of hooping, the elasticity and resistance of the same prism were measured first when hooped and then without the hoops. The latter tests have thus shown the properties which concrete retains after

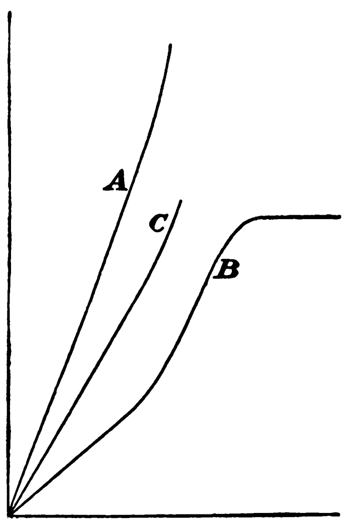

Fig. 20.

having been stressed in hooped members without having exceeded the limit set by the test load; that is, in the condition in which almost all structures are. Fig. 20 here will well serve to illustrate these properties. The ordi-

nates of the deformation curve A represent the resistances which the hooped concrete specimen offered when the shortenings represented by the abscissas took place. Curve B indicates the resistances of the core left after the removal of the hooping. Curve C has for ordinates the differences of A and B and, hence, represents the increases in resistance which the hooping has given to the concrete. It is this curve whose practically uniform inclination has given the value $n = 2.1$, which has been mentioned previously in section 10.

The phenomena illustrated by this diagram are quite different from those shown during the first application of load. It proves the predominating influence of the hooping on the concrete after the test load, while the influence of the concrete predominate before it. The direction of curvature which the curve shows up to a shortening of 0.074 per cent., which it has reached previously, corresponds well with the results found for prisms G, H, I, J, discussed in section 9. It is especially important to observe the form taken by curve B under heavy loads. Its inclination to the horizontal, which furnishes a measure of the coefficient of elasticity of the concrete proper, first increases until the coefficient reaches the value of 3,700,000 pounds for a pressure of 1,280 pounds per square inch; and then decreases very rapidly until it becomes zero for a pressure of 1,565 pounds per square inch. After having reached the latter value with a shortening of 0.095 per cent., the resistance offered by the concrete remained constant until crushing. It was not feasible to measure the shortening at the moment of rupture, but the last observations made on two opposite sides, long before the deformation had reached its limit, showed shortenings of 0.12 and 0.265 per cent., which were much exceeded before failure. This experiment proves directly that *the compressive resistance of concrete which has been hooped after having reached a certain greatest value, remains con-*

stant, at least within wide limits, notwithstanding the increase in deformation.

Exactly the same phenomena is shown by concrete-steel in tension subjected to elongations exceeding its elastic limit. This has been proved many times in the previous chapters, and it is mentioned here in order to state the hitherto unknown similarity of the properties which concrete shows under different deformations. It should, however, be noted that a possible source of error may be contained in the above results. The core of the specimen after the removal of the spirals still contained the longitudinal reinforcing rods. To compute the resistance of the concrete the compressive resistance of the longitudinal rods for each shortening had to be subtracted from the total pressure. As long as the metal was within the elastic limit no appreciable error could arise from assuming the resistance of the longitudinals to be equal to the product of their area by their coefficient of elasticity and by the amount of their shortening; but after exceeding the elastic limit the assumption was not exact. It was assumed that this pressure had a value of 100 pounds per square inch, the same as in similar specimens for which it has been measured. Whatever slight error may have been involved in the analysis of the above results, a very important fact was established. The total pressure on the prism was increased only from 70,100 to 73,850 pounds, while the average shortening increased from 0.109 per cent. to 0.194 per cent. of the length of the prism. It is thus seen that the sum of the resistances of the two materials was almost unchanged by the increase of the deformations and, since metal always shows some increase in resistance even when outside of the elastic limit, it is quite certain that the concrete did not gain in resistance during the considerable shortening of $0.194 - 0.109 = 0.085$ per cent. The resistance of 1,565 pounds which the core inside of the hooped steel showed after the removal of the hooping

following a first test load should be compared with the resistance of prisms of the same concrete without hooping, which had values of 832 to 1,170 pounds per square inch.

In the prism just discussed the preliminary compression, with the aid of the hooping, increased the resistance of the concrete by 50 *per cent.* This prism was the only one which gave such good results. The others failed, after removal of the hooping, without much shortening. This was due either to damaging the concrete when tearing off the spirals or to unequal bearing at the bases. But in such investigations a single positive fact is sufficient to illustrate the property of the material. It seems certain that this property of concrete must always produce its effects in hooped members, because numerous prisms experimented upon, without exception, stood great shortening without injury, which is essential to the development of this property.

12. Practical Rules for the Computation of Hooped Concrete Members.

The considerations discussed in the preceding sections covered the various points which had to be known to determine the values of the coefficient of elasticity and the compressive resistance of members reinforced by hoops and longitudinals. Only a small place was given to the elastic limit, as it may be raised at will to any amount desired, even to the compressive resistance, which is considerable. The value of the elastic limit is, therefore, not important for structures made of members subjected to test loads exceeding the working load. This method will be described in section 16; but for other structures the value of the elastic limit should receive attention.

The elastic limit of hooped members, as shown, under a first load evidently depends on that of the concrete, which is reached before that of the metal. It may be admitted that the shortening of the concrete is much in-

creased under higher pressures, and that, hence, the practical elastic limit is reached when the shortening attains 0.08 per cent. to 0.13 per cent., according to the character of the concrete. This will be assumed in the following. On the other hand, it will be prudent to neglect the fact, which appears from experiments, that the resistance shown by the concrete in hooped members exceeds by about 50 per cent. that of the same concrete not reinforced. For the same reason it will be well to neglect the other fact that the longitudinal rods undergo, before the crushing of the hooped members, considerable shortenings which are much greater than the elastic shortening of the metal and which, therefore, produce resistances much above the elastic limit. In members without reinforcement, on the contrary, the elastic limit cannot be exceeded. We thus arrive at the following rules:

Coefficient of Elasticity.

1. For the first load, the coefficient of elasticity of a hooped member is equal to the sum of the coefficients of the concrete, of the longitudinal rods, and of the imaginary longitudinals, whose volume shall be assumed as 90 per cent. of the hoops, or spirals.

2. For pressures less than a previous test load, the coefficient of elasticity of a hooped member is equal to the sum of the coefficients of the concrete, as increased by the test load, of the existing longitudinal rods and of imaginary longitudinals whose volume shall be assumed as double that of the hooping, or spirals.

Elastic Limit.

The elastic limit of a hooped member, for a first load, is equal to the natural elastic limit of the concrete increased by the resistance of the reinforcing as found for a shortening of 0.08 to 0.13 per cent. and computed on

the basis indicated above for the coefficient of elasticity under a first load. Every load has the effect of making the final elastic limit practically equal to the pressure due to this load.

Compressive Resistance.

The compressive resistance of a hooped member exceeds the sum of the following three elements:

1. Compressive resistance of the concrete without reinforcing.
2. Compressive resistance of the longitudinal rods stressed to their elastic limit.
3. Compressive resistance which would have been produced by the imaginary longitudinals at the elastic limit of the hooping metal, the volume of the imaginary longitudinals being taken as 2.4 times that of the hooping.

13. The Column Resistance of Hooped Concrete.

It is not sufficient for a structural member in compression to have adequate crushing resistance, it must also offer resistance to lateral flexure. The well-known formula of Euler gives the resistance N of a member whose coefficient of elasticity is E, length between "hinges" l and least radius of gyration r.

$$N = E\pi^2\frac{r^2}{l}$$

It is known that this formula is exact only for a very long member of little resistance and does not agree with results obtained on columns of dimensions met in practice.

An attempt was made to establish an empirical formula which should agree well with the results of experiments, especially those made by Hodgkinson on cast-iron columns with flat bearings; and by analogy the well-known Rankine formula was adopted for iron and steel columns with pin-bearings. In this formula, it will be

remembered, the column resistance is a direct function of the crushing strength of the metal.

Experiments made on iron and steel columns with hinged ends and reported by the author to the Congress on Methods of Construction, of 1889, and to the French Commission on Methods of Testing, of 1892, have proved that the Rankine formula is far from being exact and that it is fundamentally wrong, because it makes the column resistance a direct function of the crushing resistance only. It is quite true that these two quantities tend to become equal for prisms of very small length, but there is no necessary relation between them when the dimensions are within limits generally met in practice. And in this case, which is the only one of real interest, the column resistance depends entirely on the elastic variations. If then the Rankine formula cannot be of use for a metal of practically uniform quality, for which its constants have been determined, it is impossible to obtain from it even probable results for new materials.

The same experiments have proved that Euler's formula is exact for iron columns of any dimensions if for the coefficient of elasticity that value is introduced which it has when the column is under flexure and not, as is usually done, the coefficient of elasticity corresponding to a light load. Such an interpretation of the Euler formula does not allow of a solution for N, because it contains a value of E which is itself a function of the unknown N. But the formula can be written in the form $\frac{r}{l} = \frac{1}{\pi} \sqrt{\frac{N}{E}}$ and it can be used to determine the value which shall be given in the ratio $\frac{l}{r}$ in order that the column resistance shall have the value N. For this purposes it suffices to introduce any value N with its corresponding coefficient of elasticity. Tables giving the resistances for different ratios $\frac{l}{r}$ can thus be made.

The application of the Euler formula to concrete requires additional careful considerations. The formula is based on the assumption that a loaded column has an indefinitely small curvature and that it is in equilibrium under the action of the pressures which pass through the centers of gravity of its bases. This means that there is equilibrium between the moment of resistance due to the bending and the bending moment caused by the load, which is equal to the product of this load by the deflection. The moment of resistance consists of two components. One is represented by the sum of the increases in pressure caused in the fibres nearest to the center of curvature by the increase of their shortening. The other is given by the sum of the decreases in pressure of the fibres on the opposite side whose shortenings are relieved by the bending. The coefficient of elasticity of the column is for one component the coefficient of the material under a first load, if the column has never been loaded before, and for the other component the coefficient for a decreasing load. We have thus two coefficients of elasticity while Euler's formula contains only one. It was shown in Section 9 that these two coefficients are almost equal under light loads, and that their values continue to diverge as the loads increase until the difference becomes very great when the elastic limit is exceeded.

If the ratio of the smaller to the greater of these two coefficients be denoted by m and the distance from the neutral axis to the extreme fibre having the less coefficient of elasticity by x, and the total depth of the section be taken as unity, the following equation is obtained by equating the two components of the couple: $m x^2 = (1 - x)^2$. Whence $x = \frac{1}{1 + \sqrt{m}}$. The distance between the two forces of the couple being always two-thirds, independent of the value of x, their moment is proportional to either of them. It results from this formula, using

the same highest value for the coefficient of elasticity, and $m = 1.0$, 0.50, 0.25 and 0.09, that the moments are proportional to 1.0, 0.68, 0.44 and 0.21, respectively. When m is near to unity the average value of the coefficients E can be introduced in Euler's formula.

By plotting the results of the experiments on prisms G, H and I, given in Section 9, it is seen that for the pressure of 3,270 pounds per square inch the coefficient of elasticity varied between 2,850,000 and 3,570,000 pounds, under the first loading, and between 5,690,000 and 7,110,000 pounds during the unloading. If in the above formula the lowest values of these coefficients be introduced, which have a ratio of about 0.5, the value of E found by this method to be put in Euler's formula for a resistance $N = 3{,}270$ pounds per square inch is $E = 3{,}900{,}000$ pounds. The value 0.0091 is thus determined for $\frac{r}{l}$. For cylindrical members this value of $\frac{r}{l}$ corresponds to $1 = 27$ diameters. In other words, in order that a concrete column should have under a first load a column resistance of 3,270 pounds per square inch the length of the column, center to center of "hinges," must not exceed 27 diameters. Higher values than 3,270 pounds can be obtained under the first load only by appreciably reducing the length of the column, since above this value the coefficient of elasticity rapidly decreases. It should, however, be remembered that the resistance also increases rapidly with a decrease in length, it being proportional to the square of $\frac{r}{l}$.

Much higher resistances can be found for members which have been preliminarily subjected to a sufficient test load. Thus for a load of 6,400 pounds per square inch an average coefficient of elasticity exceeding 4,830,000 pounds can be expected, and the Euler formula shows that to obtain the above high value the greatest length

of column must not exceed 22 diameters. Hinged columns are really seldom met in practice and the lengths of the columns could, therefore, somewhat exceed the limits named. Concluding, it may be said that it is not the fear of flexure which makes it inadvisable to use for hooped concrete 3,270 pounds for the first load and 6,400 pounds per square inch after preliminary test loads. It is self-evident that as these pressures represent the upper limit of values, a margin of safety should be left for the working loads.

14. Factor of Safety.

The preceding discussion has brought out the mechanical properties of hooped members in compression, but it has not established direct rules for the allowable working load with the provision of a sufficient factor of safety. There is no universally accepted principle for the determination of the factor of safety. Sometimes its value is based on the ultimate strength of the member and sometimes on its elastic limit. Concrete reinforced longitudinally only has no definite elastic limit, or, rather, this limit almost coincides with its ultimate strength. In hooped concrete, on the contrary, the elastic limit differs materially from the ultimate strength, as in ductile metals. The working stress which will be allowable for each system will, therefore, differ much according to the method used in its deduction.

Some possible faults which the factor of safety must insure against are common to all structures, as, for example, probable inaccuracies, possible increase in the loads or in other external forces, etc. The liability to rusting is the same, and it is very small for all well-studied concrete-steel types. The direct effect of shock is less to be feared for hooped than for longitudinally reinforced concrete, since the amount of kinetic energy a member can absorb without breaking is represented by the area con-

tained between the deformation curve and the axis indicating the shortenings, and this area, when plotted, is greater for the first type than for the second, as shown by Fig. 19. On the other side, the indirect results of shocks, vibrations, have the less effect on structures the greater the mass is, and longitudinally reinforced concrete, necessarily, requires more mass; but this is really of small importance, since the hooped concrete structures have enough mass for all practical purposes. These considerations do not show much difference between the two types. It is only in approaching the stress acting in a structure that this difference becomes apparent.

It is known that the coefficient of elasticity of concrete varies with its mixing, especially with the amount of water used and the thoroughness of the tamping. The stresses are for the same deformation, proportional to the coefficients of elasticity, and they may, therefore, show much difference between different points of the same section, even when subjected to symmetrical deformation. Generally, the deformation of members computed to act symmetrically may, for many reasons, be irregular. Even in the most favorable cases, as in hinged arches or simple beams, much depends on the transverse connections, rigidity of connections, irregularity of work, etc. Frequently these simple structures will not be adopted because hinges are delicate things for structures of great weight and because the engineer does not wish to miss the advantages of a continuous structure. But the importance of the elasticity of the material is greater for fixed arches and continuous beams, because the deformation of the centers during construction and unequal settling generally cause stresses which are not provided for in the design of the structure and which may at some points considerably increase the total stress. Thus there are two kinds of stresses to be considered. The primary stresses are caused by the external forces in a structure of given form, are

independent of the coefficient of elasticity, and depend only on the resistance of the material for the support of the structure. The secondary stresses are due to the irregularities of construction and the deformation of the original form. These are, evidently, the more dangerous, the less ductile the material is, and the sooner it breaks under deformation. Only the first kind of stresses can be anticipated and computed, but the other stresses must be provided for by a proper selection of the margin of safety.

A glance at specifications for steel bridges and buildings generally used in American practice will convince any one that for static loads the greatest allowable unit stresses are about one-half of the elastic limit. This gives a real factor of safety of 2, as the elastic limit must not be exceeded, if crippling of the structure would be avoided. It is impossible to evaluate accurately the factors of safety which are used for masonry structures, extremely different as they are, but it is known that they are considerably above 2. No doubt, their high values are partly due to the fear of poor work. They are justified, however, when one remembers how small the deformations are which masonry can sustain, and how frequently stones scale off and deteriorate.

No such high factors of safety would be used for ductile materials having an elastic limit well below the ultimate strength.

Observations on the deformations of steel structures have shown that the elastic limit of the metal is the safe limit for structures, and that it is sometimes reached at some points of the structure without in any way endangering the structure. On the one hand, the exceeding of the elastic limit is the result of insufficient resistance, which, when due to excessive loading, can only become worse, and indicates approaching danger; on the other hand, it is the means by which the ductile materials free themselves of danger, if it is a result of excessive deformation.

These considerations lead to the conclusion that the required margin of safety is greater for longitudinally reinforced concrete, which is brittle, than for the hooped concrete, which can undergo very much greater shortening without failure.

There is no doubt that concrete-steel has a decided advantage over steel in structures in its resisting capacity to atmospheric influences. Concrete exposed to air or brackish water can be considered as having an indefinite life. Certain Roman mortars are in an absolutely good state to-day, and there is no indication that Portland cement gives worse results. Much evidence has also been brought out showing that metal embedded in an ordinarily good concrete undergoes no changes. The recent experiments of Breuillé, published in The Engineering Record of September 20, 1902, prove that in certain cases heavy water pressures acting on intensely stressed concrete-steel change the concrete and then the ferric layer which surrounds the reinforcing steel, and destroy the adhesion, if the pressure is alternately put on and taken off. This is a very important fact, which seems to prohibit the use of concrete-steel for water reservoirs. But there is no similarity between the conditions under which these experiments were made and those of ordinary structures exposed to the air and the rain. Countless examples of this character are in existence at an old age showing no change of quality.

As to dynamic effects, concrete-steel structures show advantages. This was proved by numerous experiments, the results of which M. Rabut communicated, in 1902, to the French Academy of Sciences. At the same time he established the fact that, due to the undeniable superiority of concrete-steel structures as to the intimate connection of their parts, stresses caused by concentrated loads are better distributed than in steel structures. Hooped concrete, with properly reinforced joints, has no weakened section, and

the remarkable ductility shown by the experiments can be counted upon to exist at any point of a member. It may suffice to state that a riveted steel member $5\frac{1}{2}$ inches in diameter will certainly not bend without fracture to a radius of curvature of 23.6 inches, as a hooped concrete member of the same dimension has done without breaking and still maintaining a degree of strength, as proved by final tests. While considering all these advantages of hooped concrete, its disadvantages should also be stated.

It is correctly claimed that the quality of concrete-steel structures depends to a large extent on the care exercised in their construction and that it is impossible to control the construction at every instant. Bad work can much reduce the resistance of compression members and the adhesion of the concrete to the metal, without which no reinforcing of concrete is effective. The importance of this serious objection is diminished if the completed structures are tested by loads appreciably higher than their working loads. But the objection still holds true, and it has added much to the weight of the author's proposition to build structures of single members or blocks which have been made in shops under continuous inspection and which will be tested, before being put in place, for much higher stresses than may occur in the finished structures. It is important to note that in hooped-concrete members the influence of the quality of the concrete is, after the test load has been applied, quite secondary, and that their elasticity and column or crushing resistance depend almost exclusively on the reinforcing metal. It is easy to inspect and check the dimensions and sizes of the metal skeleton of each member before concreting. With the above methods of construction there will still result, in many cases, considerable economic advantages for hooped-concrete construction. The most important is that there need be no apprehension of bad work either in the joints, which can easily be made excessively strong, or in the

main sections of the members manufactured and tested in the shop.

The logical conclusion of the above discussion would be the adoption for hooped concrete of a factor of safety smaller than that generally used for metal structures, which varies between 2 and 2.5 on the basis of the elastic limit and of the column resistance. But concrete-steel, especially hooped concrete, has an indisputable drawback; it represents a novel method of construction which has not stood the test of years of experience. For this reason the author proposes a factor of safety of 3 to 3.5 for hooped-concrete structures. It should be noted that the factor of safety does not generally exceed 2 in concrete-steel structures in bending where the iron is stressed in tension from 11,000 to 14,000 pounds per square inch, and sometimes still more, while the elastic limit does not exceed 23,000 tc 28,500 pounds. Considering this fact together with the considerably increased reliability as resulting from the proposed method of construction, the author believes his propositions to be more than safe. Referring to the column resistances given at the end of section 13, it will be seen that the adoption of a factor of safety of 3 to 3.5 will stress the concrete from 950 to 1,100 pounds per square inch for structures concreted in place and from 1,850 to 2,000 pounds for structures built of blocks previously tested under sufficient loads.

15. Types of Hooped Concrete.

The good results of hooped concrete depend on the proper arrangement of the hooping. It is evident, at the start, that the hoops must be well locked and that they must not open under the internal pressure of the concrete. If the inclosing reinforcement is made of other material than metal, the stresses which it produces in the concrete must not injure its resistance to compression endwise, which is its main stress. The experiments reported earlier

in this chapter have also proved that the hoops must be near to each other to give to the concrete a satisfactory coefficient of elasticity and column resistance. To duly appreciate the degree of regularity required for the spacing of the hoops a distinction should be made between the crushing resistance and the elasticity on which the column resistance depends. It has been shown that hooped prisms always have a considerable excess of crushing resistance. It will, therefore, not impair the crushing resistance if the spacing of the hoops is not very regular. Of course, too much irregularity should be avoided and the omitting of a hoop, for instance, would prove dangerous. For the elasticity and column resistance the regularity of the spacing must not necessarily be perfect. The deformations measured over sufficient lengths depend on the average spacing for the modification of the column resistance.

Hoops of two completely different types may be used. One type consists of independent hoops, the other of continuous helicoidal spirals made of wire or rods as long as possible. If independent hoops are employed each one must be locked so strongly as not only not to open under the internal pressure of the compressed concrete, but also not to show any appreciable deformation in its connections. Practically it is impossible to connect the countless hoops which an important structure would contain. This has never even been attempted and it is generally limited to giving the ends of each rod an excess of length which overlaps and locks by the adhesion of the metal to the concrete. The ends can be tied together if the wire is of small gauge, but this is not good practice in all cases and is almost impossible with thicker rods. The introduction of the adhesion to the concrete causes a stress which can in nowise be neglected and which is produced near the circumference of the compression members where the pressure and the tendency to flexure

require all the resistance that can be afforded. In addition it must be remembered that the regularity of spacing of the independent hoops is dependent on the care and attention of the workmen, on which it is impossible to count.

These inconveniences are avoided by the use of long wires or rods bent into helicoidal spirals. Drawn wires of great resistance can be obtained up to $\frac{1}{2}$-inch diameter and steel bars $\frac{5}{8}$-inch thick can be obtained in coils 160 feet long. The thickest bars that may be required for hooping can be bought in coils at least 80 to 100 feet long. With such lengths 10 to 40 spirals can be made continuous. To insure the transmission of the tension from one bar to another it is sufficient to embed between the last spirals of one bar the first spiral of the next and to curve in toward the center of the section the ends of the bars bent into crooks. The additional stress thus caused in the concrete acts in the central portion of the concrete where, being supported by the pressures acting in every direction and having a considerable excess of resistance, there is no danger. Experience has plainly proved the efficiency of this procedure. It could not be well used for independent hoops where the too numerous ends would crowd the section. When spirals have to be made for given prisms, an endeavor should be made to find the diameter and pitch which the metal, when left to itself, will retain, owing to its elasticity. Experience has shown that this is not difficult. The spirals once made and checked, no important errors can occur while putting them in place and concreting.

It would seem that, since the elastic efficiency of the hooping increases with the closeness of the hoops, at least within certain limits, the ideal will be reached by the use of continuous tubes; but such is not the case for several reasons. The external layer of concrete which is necessary for the protection of the tube will crack off

as do all covers applied to surfaces subject to high stresses. The tube would generally be riveted and pierced by numerous holes which would reduce its resistance not only by reducing the section, but frequently also by punching defects. In addition to these practical objections a fundamental objection must be taken into account. It has been seen that the metal of the hoops can resist the considerable deformations of the hooped prisms because its elongation is only a small fraction of the longitudinal shortening. It is far below its elastic limit when the reinforcing parallel to the axis already exceeds it. The longitudinal shells would be in the same condition as the longitudinal reinforcing rods. They would be stressed the same as these and would spread laterally the same as the concrete itself and would thus not be able to prevent its swelling. Concluding, it may be stated that independent rings show serious difficulties which increase with their closeness and the sizes of the iron used for them. Continuous shells, on the other hand, have little efficiency and are exposed to atmospheric action. The best type of hooping thus appears to be helicoidal spirals of suitable dimensions.

For the reported experiments a single volution was used which was cut by a plane perpendicular to the longitudinal axis in a single point. Two modifications of the same may be used. One consists in having two or three wires, instead of one, wound in a spiral doubling or tripling at the same time the pitch of the screw so as to maintain the same spacing of spirals. This can be done if it is not desired to rely on a single wire, though it has not been worked otherwise than by bending it in the spiral shape. The other modification consists in giving to the spirals different diameters, instead of the same for all, and having the direction of the spirals in the opposite direction. This would prevent any torsional tendency in the concrete, but torsion is evidently out of ques-

tion in the structures discussed. From the above the author concludes:

To give to concrete a high ductility and to increase at the same time to a high degree its elastic limit and crushing as well as column resistance, the hooping must form close loops and have the least number of joints. These conditions lead to the use of helicoidal spirals combined with longitudinal rods forming a continuous skeleton which resists efficiently the transverse swelling of the concrete.

16. The Method of Manufacture of Compression Members.

Hooped concrete, the same as concrete-steel generally, will take many forms and have many applications which will determine the methods of its construction. Nothing general can be said on this subject, and the following remarks apply only to columns or compression members of sections approaching to circles and intended to resist pressures endwise.

The importance in such members of elastic symmetry, that is, of the equality for the values of the coefficient of elasticity and the elastic limit at different points of the same section is easily understood. The lack of symmetry causes curving and invites flexure. But it is practically impossible to obtain elastic symmetry in members concreted while lying longitudinally, as it is seen at once that there must exist differences between the upper and lower layers as to tamping and distribution of water, cement, sand, and stone. By examining and sounding with a hammer different parts of a member appreciable differences can almost always be noticed in the compactness. This consideration tends to the adoption of vertical moulding for members compressed endwise; but it should be noted that this does not apply to beams whose horizontal layers are subjected to different stresses. For them

the sum of their moments only is of importance and not the distribution over the section.

Vertical moulding has also other advantages. It tends to place the stones with their flat sides perpendicular to the direction of the pressure, the same as in well-built masonry. Horizontal moulding, on the contrary, places them in the most unfavorable position. The concrete being better confined in vertical moulds than in horizontal, tamps more quickly and much better. In members reinforced by hoops and longitudinals pressing against the inner surface of the hoops the whole central portion is free for vertical ramming, and the tamping tool can strike the concrete without shaking the reinforcing and disturbing the previously tamped portions. If members which are to be built in a different position are moulded vertically they get the advantage of being made in improved moulds provided with stops for the spacing of the spirals and in a covered shop protected from the rain, which is dangerous during moulding, and the sun, which is not less dangerous during the early time of setting. Thorough inspection in a shop is easy and only a small number of selected workmen are required. For complete safety all members should be carefully examined and their quality judged by sounding with a hammer, which gives valuable information to the experienced man, and by weighing from which the density of the member could be computed and which would add additional information on the character of the concrete. Indeed, for a concrete of a given proportion the elastic limit and the resistance vary in the same sense as the density and are in proportion to the intensity of the sound. Members not up to the standard could then be rejected.

Vertical concrete members can be taken out of the moulds at the end of six days. The same moulds can then be used over again many times. Beginning sufficiently early with the making of the members, they will

acquire all the desired resistance before being put in place. The members will be put during their hardening in conditions which are best fitted for them, in moist air or even in water. On the other hand, there are some disadvantages attached to the shop process and the vertical moulding. It may be feared that the tamping of the central portion does not compact the concrete in the annular space between the hoops and the external surface though experience shows that the cement and the finer sand and gravel flow into this space and fill it to a satisfactory degree. But it has been shown by the experiments reported in this chapter that the central portion only is of real importance as to resistance. The external layer only protects the metal from atmospheric influences. After the removal of the moulds it is easy to see whether the metal will be well protected and, if required, the surface may get a finishing touch. Visible and easily repaired faults have only a relatively small value and no others can exist in hooped-concrete members moulded vertically. The second disadvantage of shop moulding consists in the required handling and transportation to the work. The increased cost due to this may in whole or in part be compensated by the saving of the forms used on the work. The third and, apparently, most serious objection of shop moulding is the inconvenience of assembling. Careful examination shows that this objection is not serious for hooped concrete.

It is impossible to foresee with certainty the ways and means which practice will develop and adopt for the assembling of the members, but some indications may be given which, while open to improvement, will assure safe results. In consideration of local stresses of high intensity which may be induced at the joints, the ends of the members will be reinforced by additional hoops of finer wire than that used for the regular hoops. The joints will thus be strengthened to any desired degree in-

dependent of the other parts of the structure. To give an idea of the efficiency of hooping after ramming it is sufficient to state that the prism tested at Quimper, whose crushing resistance, as given in section 5, had the high value of 10,500 pounds per square inch, consisted of a mortar cylinder three months old which was then hooped and covered by cement and tested ten days later.

It will generally be useless to provide for the continuity of the longitudinal rods in hooped members since their duty is mainly to form with the hoops a network which will prevent the swelling of the concrete and they fulfill it perfectly by simply stopping at the joints. Cases may, however, occur in which it will be prudent to insure transmission of tensile stresses in the longitudinals which may be caused by bending moments. In such cases it will be easy to embed in the concrete at the ends of the rods, while moulding, iron tubes which will come opposite each other and in which metal rods of the same section as the longitudinals will be placed and grouted by cement. Or the longitudinal rods on the end of one member may be left projecting and some tubes be placed in the corresponding end of the other member. Joints made in this way will not, as in other structures, be weak joints. If, due to settling of centers, irregular filling of joints or any other cause, unequal distribution of stress occurs, the great ductility of hooped concrete and the enormous excess of resistance at the ends of members will exclude all danger.

Concrete reinforced by longitudinal rods without hoops and made in single members will offer difficulties similar to masonry, at least when no hoops are used near the ends. But even when laid in place without joints the longitudinally reinforced concrete in long compression members has a defect which has not as yet attracted sufficient attention. In this type of construction the longitudinal rods make up a very important part of the resistance and the transmission of pressure through them must be per-

fectly assured. Their connection is generally made by placing the adjacent ends of the rods in very short tubes. These ends are cut very irregularly and are not generally in bearing contact except by some projection which yields under the pressure and allows the abutting ends to approach, producing thereby local deformations in the concrete whose magnitude and effect cannot be foreseen.

To avoid this objection reliance is placed on the cement which penetrates into the tubes during the ramming, but this is quite uncertain and it is also to be noted that badly-rammed cement will not be able to resist pressures of 11,000 to 17,000 pounds per square inch which the rods transmit to it. Sometimes another method is used. The rods are straddled over each other and reliance is placed on the adhesion of the concrete for the transmission of the pressure. This method is good for hooped concrete because the rods have here a secondary function only and are embedded in a concrete which is protected from all danger by the hoops. But the same method is objectionable in structures in which the longitudinals perform a main function, and in which there is nothing to prevent breaking up and crushing the concrete which is highly stressed by the transmission of the pressure.

Numerous reasons thus render advisable the making of hooped-concrete members under cover, protected from weather and by selected workmen under a continuous inspection of responsible foremen. Referring to section 9 it will be seen that to obtain the greatest efficiency from this type the members must, before being put in place, be subjected to preliminary pressures of higher intensities than they will be called upon to resist in the structure. Independently of the important guaranty which the preliminary testing of all members of the structure gives and the convenience of rejecting doubtful members, the preliminary testing results in a great increase of elasticity and column resistance.

Responsible contractors, to whom important structures are let, will not find any difficulty in providing the necessary testing apparatus and accessories. This will be cheaper than may be expected because the shortenings which will have to be produced in the tested members are quite small, less than 0.04 per cent., and the stress required is so small that a cylinder and piston of a hydraulic press with a hand pump worked by one or two men will be sufficient to test very strong members. Neither motors nor accumulators will be required. The methods of manufacture and erection which have been discussed in the above are similar to those used on metal structures and guarantee the good execution required for works of some magnitude. Of course, small structures will not require these methods, as has been proved by many existing structures.

17. The Influence of the Character of the Materials.

Most builders use for the concrete the same proportions, 500 pounds of cement to 1.2 cubic yards of sand and gravel, which yield one cubic yard of concrete put in place and tamped. In floors and beams and other members subjected to bending, the tensile resistance of the concrete is not taken into account. There is, therefore, little advantage in improving the character of the concrete in the parts in tension. It seems that such is not the case in the compressed portions where the concrete supplies the most, if not the whole, of the resistance. But the two following points should be considered. If the concrete for the space between the main beams is made richer, this will be done in order to decrease their thickness, but the moment of resistance decreases as the square of the thickness, while the volume of the concrete decreases as its first power only. The cost decreases still more slowly than the volume because certain expenses remain the same, as,

for instance, that of the moulds. The conditions are similar if not identical for the main beams. Finally, it is known, and the author has proved it by exact data, that the cracks which appear in tension members exposed to dry air and especially to the sun while under the partial load which they carry during their hardening period, are the more important the richer the concrete is.

Neither of these reasons holds true for compression members, except the first in the quite rare case where the members are so long as to require provision against flexure. On the contrary, for compression members which will not be tested in advance, there is a decided reason for richer concrete. For the first load imposed on concrete the elastic limit, which has a considerable influence on the column resistance, increases in a high degree with the proportion of cement when the same is increased from 500 to 1,000 pounds. This fact is shown by all of the author's experiments, and an example can be seen by comparing the deformations reported in section 9 for three prisms of 1,000 pounds and one of 500 pounds of cement.

No doubt the importance of the initial elastic limit decreases after the testing of the members before putting them in place but it does not vanish altogether. This method of construction will only be used when high resistances with all possible guaranty will be desired, and the small increase in cost of a richer concrete will be willingly paid. The whole difference is not more than 10 to 20 per cent. of the cost of concrete. The author believes that in any case for members subjected to great stresses 750 pounds and frequently 1,000 pounds of cement per cubic yard of concrete laid in place should be used for a mixture of sand and gravel so balanced as to have the least amount of voids.

The choice of the metal to be used also deserves attention, and it should be here remembered that the longitudinal rods are compressed beforehand by the shrinking

of the concrete and reach rapidly their elastic limit, while the hoops must be compressed before taking tension, and that their stresses are small when the longitudinal reinforcing is already at the end of its resistance. The elongations of the hoops increase, therefore, about three times as slowly as the shortenings of the longitudinals. The experiments show that the greatest shortenings which will have to be considered, not for working stresses but for dangerous limits, do not exceed 0.2 to 0.3 per cent., and it is certain that the corresponding elongations of the hooping metal do not reach 0.07 to 0.1 per cent. and thus remain below the elastic elongation of common wrought iron and soft steel. The elastic limit and the ultimate resistance of the hoops is, therefore, not important and their effect depends solely on the coefficient of elasticity. But it is known that the coefficient of elasticity of soft steel is on the average 10 per cent. higher than that of wrought iron, and that for the high carbon steels it does not have any higher value. Drawn wire will thus for hooping be inferior rather than superior to bars. These considerations lead to the use of high-carbon steel for the longitudinal reinforcing and of soft steel or wrought iron for the hoops, according to the cost of the metal. The rolling into spirals is facilitated by the use of malleable metals.

It is now understood why the author has not attached much importance to the fact that the hooping of the tested prisms was made of drawn iron wire. No doubt, with bars, the ultimate resistance would not have shown such an enormous and useless excess of crushing strength, but the interesting period of the deformation would have been exactly the same, and the conclusions, which have been based solely on facts observed during this period, would not have been different.

It may be concluded that for hooped concrete intended to resist high pressures it is proper to use concrete con-

taining 750 to 1,000 pounds of cement per cubic yard of concrete laid in place, and to use high-carbon steel for the longitudinal reinforcing and wrought iron or soft steel for the helicoidal spirals which form the hooping. Generally it will suffice to make the metal between 1 and 2 per cent. of the volume for the longitudinal reinforcing, and between 2 and 3 per cent. for the hoops. This will give a high ductility and a resistance which will exceed all requirements.

18. First Cost.

The question of first cost is a very delicate one, and to find a complete solution for it long discussions are required. The author will only attempt here to furnish some first thoughts on the subject. The study of first cost must lead to a comparison of the cost of the different types of construction, among which the engineer must choose. Hooped concrete must evidently be compared to concrete-steel as ordinarily reinforced and to riveted steel.

If the structure considered is to be such as can be built of concrete-steel of any of the old types the advantages which the substitution of hooped concrete under the most economic conditions will offer will have to be considered first. Then investigation should determine the desirability of adopting the more or less expensive methods which can be applied in the making of hooped concrete. For concrete of the same proportions and same grade and percentage of metal, hooping will give much more resistance than longitudinal reinforcing, and will at the same time modify the fragility to a very high ductility, which will allow of a reduction in the factor of safety. It does not seem that the cost per cubic yard containing the same materials should appreciably differ for the two types of construction.

There are, however, two reasons which might outweigh the general advantages of hooped concrete and lead to the

adoption of the other type. The first of these reasons is that it is somewhat inconvenient to introduce for the columns a method of construction different from that used for beams, and to which the workmen are not accustomed. The second reason is that for columns or supports carrying light loads the least section is generally fixed by practical considerations and is frequently sufficiently strong with any kind of reinforcing. It is the shop manufacture of hooped-concrete members that must be mainly considered. It may be made in big shops with the special care and economy resulting from the employment of skilled labor and mechanical means. Columns and other members of hooped concrete with longitudinal reinforcing rods projecting out at the ends, so as to assure the connection between them and the beams or other parts of the structure, of high efficiency could then be manufactured.

More difficult is the comparison of hooped concrete to riveted construction. It will be limited to some indications of the cost of the materials of which the hooped concrete is made and the additional cost of its making. The cost of the hooped concrete in place so determined will then be compared to the total cost of the steel construction of the same strength. It will then be seen whether the difference in cost is sufficient to cover the handling, erection and falsework required for hooped concrete as well as the contractor's profit. The fact must also be considered that the longer the spans the heavier will be the weight of the hooped concrete as compared to steel, and that the fixed load of the structure will, therefore, be proportionately increased and its required resistance will have to be proportionately higher. But it will be found on closer examination that the difference in weights does not have the importance which it may seem to have at first sight. On the other hand, it should be considered whether the hooped-concrete structures do not have the advantage over the steel structures as to maintenance,

length of service, solidarity of parts, and, for short distances, rapid absorption of vibrations and resistance to impact.

To make a rough comparison riveted construction of ordinary soft steel will be assumed. It can be bought for 3 cents a pound, which is a low value for present prices. An allowable working stress of 10,000 pounds per square inch will be assumed on the steel. The gross section of the members will exceed the net section by 10 to 20 per cent., and, if in addition the weight of rivets and details will be considered, the allowable unit stress of 10,000 pounds is reduced to 7,500 pounds per square inch of gross section as far as weight is concerned. For hooped-concrete, as given in section 14, a unit stress of 950 to 2,150 pounds may be allowed, according whether the concrete has been molded in place without extra care and reinforced with a low percentage of metal or whether it has been made in the shop with extra care and strict supervision of a good concrete and a higher percentage of metal, and has been previously tested above its working load.

Omitting the cost of the moulds, staging, etc., the cost of the concrete may be taken at $10 per cubic yard, which is a high figure and is taken to be on the safe side. For the same reason the higher unit pressures mentioned in the above as being allowable for hooped concrete will **not** be used and the following will be assumed:

Allowable pressure in pounds per square inch	900	1,350	1,800
A block or short column one square foot in section will carry a load, in pounds, of...	129,600	194,400	259,200
A steel member of same resistance at 7,500 pounds per square inch will have a section, in square inches, of...............	17.3	26.0	34.6
It will weigh, per linear foot, in pounds...	59	88.5	118
At three cents per pound, it will cost, per linear foot	$1.77	$2.66	$3.54

Adding to the cost of the concrete the cost of the reinforcing steel, the cost of the hooped-concrete members will be found. The percentage of the metal varies between 3 and 6 per cent. of the concrete, which gives 400 to 800 pounds of steel per cubic yard at a cost of $12 to $24. The total cost of the hooped concrete will then be $22 to $34 per cubic yard. Per cubic foot it will be $0.82 to $1.26. Comparing this to the above table it is seen that more than a sufficient margin is left for the cost of moulds, scaffolding, etc.

19. Conclusions.

Hooped concrete has a high ductility, its crushing resistance is very high and exceeds the sum of the resistance of the concrete used for it, the resistance of longitudinal reinforcing rods stressed up to their elastic limit, and the resistance at the same rate of imaginary longitudinals representing a volume of 2.4 times that of the hooping metal. If the concrete were very poor and had only the cohesion strictly necessary to prevent crumbling away between the spirals, its resistance consisting of the two latter elements would still be very great. The danger due to poor work in concreting which forms an objection to concrete-steel constructions is hence almost altogether avoided by the use of hooping. Under the first load the hooping has much less influence on the coefficient of elasticity than on the crushing resistance, but even for the most ordinary concretes the value of this coefficient is sufficient to prevent flexure under high loads in structures having members of usual sizes. When a hooped-concrete member is subjected to a test load its elastic limit increases up to the value of this load, and likewise the coefficient of elasticity under a heavy load is increased in a high degree, and the more so the less satisfactory its value has been previously.

The concrete is thus improved to any required degree

by the hoops which, due to their mode of action and coefficient of efficiency, 2.4, can produce a high resistance without being much stressed themselves. The adhesion of the concrete to the metal which has such an important effect on members in bending is almost of no importance in hooped concrete in compression. It is called into action at the ends of the spirals only, which are bent in toward the center of section where its resistance is much assisted by the considerable friction developed in the core by the strong pressures exerted on all sides. To obtain this result it is necessary to space the hoops closely enough to each other to prevent the crumbling of the concrete between them. This object is attained, without spacing the hoops excessively close, by placing against the inside surface of the hoops longitudinal bars which in their turn also increase the tensile resistance.

The necessity of having the hoops sufficiently close to each other leads to making them of wire or bars bent to spirals and embedded in the concrete near the surface. The reduced number of joints is made by simple overlapping of the adjacent spirals and bending their ends into the hooped core. The strength of the joints is thus made equal to the strength of the members. The spirals are made beforehand, and it is, therefore, easy to check their dimensions before concreting and to make sure of the high character of the reinforcing which, as has been shown in the above, will suffice to secure the safety of the member even if the concrete should turn out of a lower grade than could have been foreseen. By decreasing the spacing between the spirals at the ends of the members a local excess of crushing strength can be obtained of any desired amount and also a considerable ductility. It is also easy to assure the transmission of tensile stresses from one member to another, and practical methods have been pointed out showing how to attain this result in a simple manner. The serious objections which are attached to

joints in masonry and usual concrete-steel constructions can thus be entirely eliminated.

It appears that for structures for which it is desired to obtain the highest resistance and safety together with the least weight the method of building by members made in the shop will have the following advantages: Elastic symmetry; execution of the work under supervision in well-prepared moulds by a few experienced workmen; continuous inspection and protection from the sun and the rain; possibility of keeping the members in water or in moist air to prevent cracks; opportunity to sound with the hammer, to weigh and even to test all the members, and to reject those below the standard; beginning the making of the members at the starting time of preliminary operations; putting the members in place after they have been hardened. Where the members will be tested they will greatly improve in elasticity and column resistance. The deformations after the removal of the falsework will be much reduced, even with considerable working pressures. The necessary rules to compute the resistance and elasticity of a hooped member of any proportions have been given.

The high ductility of hooped concrete, its resistance to impact and atmospheric influences and the general solidarity of the parts in concrete-steel structures logically lead to the adoption for hooped concrete of factors of safety in regard to the elastic limit and column flexure below 2 or 2.5 as are generally used for steel structures. But because of the novelty of this type of construction 3 to 3.5 has been proposed, even indicating the probability of beginning with factors of safety still higher. The reasons which sometimes prompt the use of smaller proportions of cement for concrete in tension or bending do not hold for compression members, and it is proper to use richer concretes for these members. It is advantageous to make longitudinal reinforcing rods of the highest steel that can be used

without trouble. For the hooping, on the contrary, wrought iron or soft steel are perfectly suitable.

Of two concrete members made of the same quantities of the same materials and costing the same, the one which is hooped will have more strength and a very much higher ductility than the one made according to any of the usual types. The approaching danger will in the first be heralded by the scaling off of the surface which has no influence on the strength. In the second nothing will announce the coming failure or, at best, heavy injuries which will endanger the structure will indicate it.

The cost of members made of hooped concrete is appreciably less than one-half of the cost of steel members of same resistance. In each case the fixed load of the structure must be considered, the cost of erecting the hooped-concrete members must be calculated, together with the general expenses and profits, and the different methods of construction then compared. Finally, the properties of hooped concrete appear to insure for it a place as a material for compression members between ordinary concrete and longitudinally or transversely reinforced concrete, on one side, and riveted steel members, on the other.

20. Additional Experiments.

Since the publication of the above chapter the author has made some additional experiments on hooped-concrete prisms. These experiments have all confirmed the results previously obtained, and some of the numerical values of these additional tests are given.

1. A prism 19.7 inches long of octagonal section, 4.3 inches in diameter, was made of concrete containing 840 pounds of Portland cement per cubic yard of gravel and sand mixed in the proportion of 1 to 3. The gravel had a greatest diameter of 1 inch and the sand of 0.2 inch. The spirals were made of iron wire 0.17 inch in diameter wound around a cylinder of 3¾ inches diameter with a

maximum pitch of 0.7 inch. Eight longitudinal reinforcing rods of the same wire were added. The total sectional area of the prism was 15.5 square inches. The sectional area of the core inclosed by the spirals was 11.16 square inches. The prism failed by bending as a column, without rupture, under a load of 142,000 pounds, corresponding to a pressure of 9,150 pounds per square inch of initial section and 12,700 pounds per square inch of the section of the core, which alone remained intact under the high pressures. The first cracks in the external layer of concrete surrounding the spirals appeared under a pressure of 3,420 pounds per square inch. The shortening of the prism was not measured at this stage, but was 0.12 per cent. under a pressure of 286 pounds and increased to 2.4 per cent. of its length when the last measurement was taken before failure. An identical prism made of the same concrete, but without reinforcing, gave a resistance of 2,240 pounds per square inch.

2. A prism of the same length, 19.7 inches of octagonal section, 12.6 inches in diameter, was made of the same concrete as No. 1. The spirals were of iron wire 0.39 inch in diameter and wound around a cylinder of 10.6 inches diameter with a maximum pitch of 1.46 inches. Eight longitudinal rods of rolled iron 0.59 inch in diameter were added. The total section of the prism was 131 square inches. The sectional area of the core inclosed by the spirals was 88.5 square inches. The prism crushed under a total load of 820,600 pounds by the rupture of one of the spirals. This load corresponds to a pressure of 6,260 pounds per square inch of initial section and 9,280 pounds per square inch of core section. The first cracks appeared under a pressure of 285 pounds per square inch. The shortening of the prism before rupture amounted to 4.2 per cent. of its length. No comparative identical concrete prism was made with this one. Examination of the concrete of this prism showed that it was not of as good a

quality as prism No. 1. The iron wire of 0.39 inch diameter, which formed the spirals, had, of course, a less resistance per square inch of section than the wire of 0.17 inch diameter. It may be stated that the least results obtained exceeded the values as computed according to the rules given.

3. Though the fact that hollow prisms of hooped concrete will not resist pressures well and will crush on the inner side could have been foretold, it was found to be necessary to verify it by experiment. Two cylinders of 23.6 inches length and an external diameter of 7.1 inches were made. The diameter of the hollow space was 4.9 inches for one cylinder and 4.5 inches for the other. The resistance was found to be 2,580 pounds per square inch of concrete section for the first cylinder and 2,290 pounds for the second. The resistance of an identical concrete prism without reinforcing was 2,404 pounds per square inch. It was thus proved that hooped members must be of solid formation or have small openings only.

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